HADRON PHYSICS at the 1 GeV SCALE
and its IMPACT
— with an eye on MAMI —

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Introduction
INTRODUCTORY REMARKS

• Many interesting facets/talks of/on hadron physics at GeV scales

• Here: concentrate on precision physics & its impact for MAMI

• Guiding theme: isospin violation in QCD

• A little history: MAMI’s biggest success = S-, P-wave LETs → story tbc
**ISOSPIN VIOLATION**

- Isospin violation has two sources (QCD + QED):

\[ \mathcal{H}_{QCD}(x) = \frac{1}{2}(m_d - m_u)(\bar{d}d - \bar{u}u)(x) \]

\[ \mathcal{H}_{QED}(x) = -\frac{1}{2}e^2 \int dy \, D^{\mu\nu}(x - y)T(j_\mu(x)j_\nu(y)) \]

\[ \Rightarrow \text{unique window to quark masses for light quark and heavy-light quark systems} \]

- Both effects usually small and of the same size (e.g. \( m_p - m_n \))

\[ \Rightarrow \text{systematic machinery must cope with both these accurately} \]

- Chiral perturbation theory w/ virtual photons is the tool to analyse the strictures of the spontaneously and explicitly broken chiral symmetry of QCD
Isospin breaking in the $\pi N$ scattering lengths

WHY RECONSIDER IV in $\pi N$ SCATTERING?

- high precision measurements of pionic hydrogen & deuterium at PSI
  ⇒ need better control of the isospin-breaking corrections


\[ \begin{align*}
  &\text{Deuteron, no isospin breaking} \\
  &\text{Deuteron, isospin breaking at } O(p^2) \\
  &\text{Hydrogen energy, potential model} \\
  &\text{Hydrogen width, isospin breaking at } O(p^2) \\
  &- a^- \quad (M^{-1}_\pi) \\
  &- a^+ \quad (M^{-1}_\pi) \\
\end{align*} \]
NOVEL CALCULATION of the SCATTERING LENGTHS

- third order calculation in IR baryon CHPT with virtual photons
- largest uncertainty via electromagnetic LECs \( f_i, g_i \)
- pertinent loop graphs

\[ (s_1) (s_2) (s_3) (s_4) (s_5) (s_6) (v_1) (v_2) (v_3) (v_4) (v_5) (a_1) (a_2) (a_3) \]

⇒ analytical results to \( \mathcal{O}(m_u - m_d, e^2) \)
TRIANGLE RATIO

- triangle ratio $R$ vanishes in the isospin limit:

$$R = 2 \frac{a_{\pi + p} - a_{\pi - p} - \sqrt{2} a_{\pi - p}^{\text{cex}}}{a_{\pi + p} - a_{\pi - p} + \sqrt{2} a_{\pi - p}^{\text{cex}}}$$

- analytical expression including electromagnetic LECs:

$$R = \frac{m_p}{4\pi (m_p + M_\pi)} \left\{ \frac{e^2 f_2}{2} + \frac{g^2_A M_\pi \Delta_\pi}{4 F^2_\pi m_p} - \frac{M_\pi \Delta N}{4 F^2_\pi m_p} (1 + 2 g^2_A) - \frac{3 M_\pi \Delta_\pi}{16 F^2_\pi m^2_p} + \frac{M_\pi \Delta_\pi}{4 m^2_p} - B_{\text{thr}} \right\}$$

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$$\Delta_\pi = M^2_\pi - M^2_\pi 0, \quad \Delta N = m_n - m_p, \quad B_{\text{thr}} = \ldots$$
RESULTS for the SCATTERING LENGTHS

- relative changes w.r.t. the isospin limit

\[ \text{Re}\Delta a(\pi^- p) = -3.9^{+4.9}_{-7.5}\% \text{, } \Delta a_{\pi^+ p} = +6.4^{+7.8}_{-5.1}\% \text{, } \Delta a_{\pi^- p} = -0.36 \pm 0.77\% \]

⇒ sizeable effect in elastic channels through triangle graph \((s_5)\)

- small IV in the triangle ratio at threshold

\[ R = (1.5 \pm 0.2 f_2 \pm 0.03 a^- \pm 0.03 B^-_{\text{thr}} \pm 1.1 \text{LEC})\% = (1.5 \pm 1.1)\% \]

⇒ consistent with earlier results in heavy baryon CHPT above threshold

⇒ inconsistent with findings of Gibbs et al. and Matsinos at low pion momenta

- large IV in the yet unmeasured \(\pi^0 - p\) scattering length

\[ \Delta a_{\pi^0 p} = (-5.2 \pm 0.2) \cdot 10^{-3}/M_\pi \gg a^+ \] ⇒ need a measurement
THE CUSP in $E_{0+}$

- Electric dipole amplitude: $E_{0+}(\omega) = -a - b\sqrt{1 - \omega^2/\omega_c^2}$

- Cusp parameter: $b = M_\pi a_{\pi^+ n}^{cex} E_{0+}^{\gamma p \rightarrow \pi^+ n} = (3.43 \pm 0.08) \cdot 10^{-3}/M_\pi$ [data] = $3.63 \cdot 10^{-3}/M_\pi$ [c.v. CHPT]


- But: large IV in the CEX $\Delta a_{\pi^+ n}^{cex} = (-1.9 \pm 0.8)\%$

- Better th’y analysis needed

- Measure precisely the polarized target asymmetry $T \propto \text{Im}[E_{0+}(P_3 - P_2)]$

$\Rightarrow$ MAMI A2 proposal!

\[ \text{Hadron Physics at the 1 GeV scale and its impact – with an eye at MAMI – Ulf-G. Mei\ss\ner – MAMI and beyond, Mainz, March 30, 2008} \]
Electromagnetic corrections to $\eta \rightarrow 3\pi$

ISOSPIN VIOLATION & $\eta \rightarrow 3\pi$

- Isospin violation drives $\eta \rightarrow 3\pi$, CHPT analysis:

\[
A(\eta \rightarrow \pi^+ \pi^- \pi^0) = A^{(2)} + A^{(4)} + \ldots
\]

\[
A^{(2)} = \frac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2} \left(1 + \frac{3(s-s_0)}{M_\eta^2 - M_\pi^2}\right)
\]

\[
\rightarrow \Gamma(\eta \rightarrow 3\pi) \sim Q^{-4}, \quad Q^{-1} = \frac{m_u - m_d}{m_s - \hat{m}}
\]

- worked out to tree, one- and two-loop accuracy
- large unitarity corrections (FSI)
  \rightarrow can be handled with dispersive machinery or UCHPT
- many testable predictions ($\Gamma(3\pi^0)/\Gamma(\pi^+ \pi^- \pi^0)$, Dalitz slopes)
- small em corrections at $\mathcal{O}(e^2 m_{\text{quark}})$, Baur, Kambor, Wyler
- but how about em corrections $\mathcal{O}(e^2(m_u - m_d))$?
NLO CONTRIBUTIONS

- strong and em diagrams

- full propagator

\[
\begin{align*}
\text{full propagator} &= \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \text{diagram 5}
\end{align*}
\]

- LECs: some strong $L_i$ (known), some em $K_i$ (dimensional analysis)
RESULTS: GENERAL REMARKS

• neutral and charged amplitudes must be calculated separately

\[ A_n(s, t, u) \neq A_c(s, t, u) + A_c(t, u, s) + A_c(u, s, t) \]

• always compare to the strong one-loop amplitude of Gasser and Leutwyler (GL)


• EM corrections in general small (but need to be accounted for)

• corrections of order \( e^2(m_d - m_u) \) (DKM) as big (or bigger) as \( e^2m_q \) (BKW)

• Coulomb-pole at the edge of the physical region in the charged amplitude

• cusps in the neutral amplitude due to rescattering \( \pi^0\pi^0 \rightarrow \pi^+\pi^- \rightarrow \pi^0\pi^0 \)


• timely: WASA-at-COSY, CB at MAMI, . . .
AMPLITUDES for $\eta \rightarrow \pi^0 \pi^+ \pi^-$

- One-loop representation with em corrections: real and imaginary part
- uncertainties from varying the $K_i \rightarrow K_i \pm \frac{\Sigma_i}{16\pi^2}$ (hardly visible)
AMPLITUDES for \( \eta \rightarrow 3\pi^0 \)

- One-loop representation with em corrections: real and imaginary part

- Uncertainties from varying the \( K_i \rightarrow K_i \pm \frac{\Sigma_i}{16\pi^2} \)

GL  BKW  DKM
DALITZ PLOT for $\eta \rightarrow 3\pi^0$

- One-loop representation with em corrections: $\pi^+\pi^-$ cusp structures
NORM and DALITZ SLOPES for $\eta \rightarrow 3\pi^0$

- $|A_n(x, y)|^2 = |N_n|^2 \{1 + 2\alpha z + \ldots\}, \ z = x^2 + y^2$

| $|N_n|^2$ | $10^2 \times \alpha$ |
|----------|----------------|
| GL       | 0.269          | 1.27          |
| BKW      | $-0.003 \pm 0.002$ | $+0.05 \pm 0.01$ |
|          | $=(-1.1 \pm 0.9)\%$ | $(+3.7 \pm 0.5)\%$ |
| DKM      | $-0.009 \pm 0.005$ | $-0.002 \pm 0.01$ |
|          | $=(-3.3 \pm 1.8)\%$ | $(-0.2 \pm 1.0)\%$ |
| DKM(cusp)| $-0.009 \pm 0.005$ | $+0.06 \pm 0.01$ |
|          | $=(-3.3 \pm 1.8)\%$ | $(+5.0 \pm 1.1)\%$ |

DKM(cusp): region from the cusp outward excluded $\Rightarrow$ no simple polynomial fit
**NORM and DALITZ SLOPES for $\eta \rightarrow \pi^+\pi^-\pi^0$**

- $|A_c(x, y)|^2 = |N_c|^2\{1 + ay + by^2 + dx^2 + fy^3 + gx^2y + ...\}$

|       | $|N_c|^2$ | $a$      | $b$      |
|-------|----------|----------|----------|
| GL    | 0.0325   | -1.279   | 0.396    |
| BKW   | -0.0004 ± 0.0003 | -0.008 ± 0.001 | +0.006 ± 0.001 |
|       | = (-1.1 ± 0.9)%  | = (+0.6 ± 0.1)% | = (+1.4 ± 0.2)% |
| DKM   | -0.0008 ± 0.0002* | -0.009 ± 0.005 | +0.006 ± 0.003 |
|       | = (-2.4 ± 0.7*)% | = (+0.7 ± 0.4)% | = (+1.5 ± 0.7)% |
|       |          |          |          |
| $d$   | 0.0744   | 0.0126   | -0.0586  |
| BKW   | +0.0011 ± 0.0004 | -0.0003 ± 0.0001 | -0.0010 ± 0.0003 |
|       | = (+1.5 ± 0.5)%  | = (-2.2 ± 0.4)% | = (+1.7 ± 0.6)% |
| DKM   | +0.0033 ± 0.0003* | +0.0001 ± 0.0001 | -0.0038 ± 0.0009* |
|       | = (+4.4 ± 0.4*)% | = (+0.5 ± 0.6)% | = (+6.4 ± 1.5*)% |
The cusp in $\eta' \rightarrow \eta\pi\pi$

Kubis, Schneider, *in preparation*
AN INTRIGUING CUSP EFFECT

- Neutral pion–pion scattering at one-loop including isospin breaking
- Pion mass difference $\sim$ induces a cusp in $\pi^0\pi^0 \rightarrow \pi^0\pi^0$ in the threshold region

Feynman graphs

Normalized scattering amplitude

$\Rightarrow$ Long believed to be an unobservable curiosity
REOCCURRENCE OF THE CUSP IN $K \rightarrow 3\pi$

- Rescattering graph in $K^+ \rightarrow \pi^+ \pi^0 \pi^0$:
  - gives access to the $\pi\pi$ S-wave scattering lengths $a_0 - a_2$
  - $\pi^0 \pi^0$ inv. mass distribution $d\Gamma/dM_{\pi\pi}$ sensitive to $a_0 - a_2$
    
    \[
    a_0 - a_2 = \frac{0.265}{M_{\pi^+}} \quad a_0 - a_2 = 0
    \]

- Large data sample from NA 48 @ CERN, $\sim 10^8$ events

- Two-loop rescattering contributions:
  - Systematic EFT approach w/ kin. energy resummations and systematic electromagnetic corrections

\[
\]

\[
\Rightarrow \text{promising alternative to extract } a_0 \text{ and } a_2
\]

\[
\Rightarrow a_0 - a_2 = 0.273 \pm 0.005_{st} \pm 0.002_{sy} \pm 0.001_{ex}
\]
\[
a_2 = -0.065 \pm 0.015_{st} \pm 0.010_{sy} \pm 0.002_{ex}
\]
THE CUSP IN $\eta' \rightarrow \eta \pi^0 \pi^0$

- NREFT calculation to two loops
- input: $\pi \pi$ and $\pi \eta$ scattering parameters
  - $\pi \pi$ from Roy equations
  - $\pi \eta$ from CHPT w/ large uncertainties
- match EFT parameters to the
  Dalitz plot parameters from VES
  assuming isospin invariance


⇒ cusp reduces the number of events in the pertinent $s_3$ region by 8%

compare: 13% for $K^+ \rightarrow \pi^+ \pi^0 \pi^0$ and 2% for $\eta \rightarrow 3\pi^0$

⇒ should be measured at COSY and MAMI!
Gauge-invariant calculation of kaon photo- and electroproduction on the proton

REMARKS on UNITARIZED CHPT

• valuable tool for chiral SU(3) dynamics pioneered at Munich and Valencia
  Kaiser, Siegel, Weise, Oset, Ramos, Oller, . . .

• advantages:
  ★ incorporates strict (coupled-channel) unitarity
  ★ allows to go above threshold in scattering and production
  ★ allows to deal with resonances like e.g. the $\Lambda(1405)$
  ★ gives insight into the nature of resonances/bound states/. . .

• disadvantages:
  ★ gives up some of the rigor of CHPT (power counting)
  ★ crossing symmetry violated (perturbatively restored)
  ★ gauge invariance often violated (in most meson em production calcs)

• this talk: reconcile coupled-channel unitarity with gauge invariance
THEORETICAL FRAMEWORK

- BSE for $\phi B \rightarrow \phi B$ with the Weinberg-Tomozawa interaction

$$V^{b_j,a_i}(q_2, q_1) = g^{b_j,a_i}(q_1 + q_2)$$
$$g^{b_j,a_i} = -\frac{1}{4F_\phi^2} \langle \lambda^{b\dagger}[[\lambda^{j\dagger}, \lambda^i], \lambda^a]\rangle$$

- Integral equation for photo/electroproduction

$$\mathcal{M}_\mu(q, k; p) = \mathcal{M}_0^\mu(q, k; p) + \int \frac{d^4l}{(2\pi)^4} \mathcal{T}(q, l; p)iS(\psi - l)\Delta(l)\mathcal{M}_0^\mu(l, k; p)$$

- Turtle approximation for the dressed meson-baryon vertex

$$V^{b_i,a} = \not\!q \gamma_5 \hat{g}^{b_i,a} /\sqrt{2}$$
$$\hat{g}^{b_i,a} = -\frac{D}{F_\phi} \langle \lambda^{b\dagger} \{\lambda^{i\dagger}, \lambda^a\}\rangle - \frac{F}{F_\phi} \langle \lambda^{b\dagger} [\lambda^{i\dagger}, \lambda^a]\rangle$$

$$\Gamma(q, p) = \not\!q \gamma_5 \hat{g} + \int \frac{d^4l}{(2\pi)^4} \mathcal{T}(q, l; p)iS(\psi - l)\Delta(l)l\gamma_5 \hat{g}$$

$\Rightarrow$ 6 channels: $\gamma^* p \rightarrow p\pi^0, n\pi^+, p\eta, \Lambda K^+, \Sigma^0 K^+, \Sigma^+ K^0$
GAUGE-ININVARIANT PRODUCTION AMPLITUDE

- Classes of diagrams for $\gamma^* p \rightarrow KB$ in the turtle approximation

\[
k^{\mu} M_{\mu} = k^{\mu} \left( M^{A}_{\mu} + M^{B}_{\mu} + M^{C}_{\mu} + M^{D}_{\mu} + M^{E}_{\mu} + M^{F}_{\mu} + M^{G}_{\mu} + M^{H}_{\mu} \right) = 0
\]
FIT STRATEGY

• simultaneous description of hadron- and photon-induced differential XS

\[ \pi^- p \rightarrow K^0 \Lambda, K^0 \Sigma^0 \quad \& \quad \gamma p \rightarrow K^+ \Lambda, K^+ \Sigma^0, K^0 \Sigma^+ \]

• mostly S-waves

\rightarrow \text{restrict to } q^{\text{lab}}_{\pi} \leq 1.23 \text{ GeV and } k^{\text{lab}}_{\gamma} \leq 1.25 \text{ GeV (} \sqrt{s} \leq 1.8 \text{ GeV)}

• use relativistic kinematics \rightarrow \text{some contributions from higher partial waves}

• fit parameters:

\begin{itemize}
  \item meson decay constants (to account for SU(3) breaking)
  \[ F_\pi, F_K, F_\eta \]
  \item scales in the loop integrals (or subtraction constants)
  \[ \mu_{\pi N}, \mu_{\eta N}, \mu_{K \Lambda}, \mu_{K \Sigma} \]
\end{itemize}

\Rightarrow \text{not yet a very quantitative approach}
PION-INDUCED STRANGENESS PRODUCTION

\[ \frac{d\sigma}{d\Omega}(\pi^- p \rightarrow K^0 \Lambda)[\mu b/sr] \]

\[ \frac{d\sigma}{d\Omega}(\pi^- p \rightarrow K^0 \Sigma^0)[\mu b/sr] \]

Knasel et al., Phys. Rev. D 11 (1975) 1

Knasel et al., Phys. Rev. D 11 (1975) 1
• $d\sigma/d\Omega(\gamma p \rightarrow K^+\Lambda)[\mu b/sr]$
In many calculations, only $A+F$ ($WT + FSI$) is used [no parameter refitting].

$\frac{d\sigma}{d\Omega} (\gamma p \rightarrow K^+\Lambda, K^+\Sigma^0)[\mu b/sr]$ [1.628 GeV]

$\sigma_{tot} (\gamma p \rightarrow K^+\Lambda, K^+\Sigma^0)[\mu b]$ [1.628 GeV]

$\Rightarrow$ readjustment of the parameters not advisable (large effects)
In many calculations, only S-waves in the kernel & external kinematics are used.

⇒ not applicable for differential XS or polarisation observables.
Mass splittings in heavy baryon multiplets

INTRA-MULTIPLETT SPLITTINGS

- *up* quarks are lighter than *down* quarks:

  \[ m_u = 1.5 - 3.3 \text{ MeV}, \quad m_d = 3.5 - 6.0 \text{ MeV} \quad \text{[using MS at } \mu = 2 \text{ GeV]} \]

  \[ \Rightarrow \text{the more } \text{down} \text{ quarks in a state, the heavier a state in a multiplet is} \]

  e.g. \[ n(udd) > p(uud), \quad K^0(d\bar{s}) > K^+(u\bar{s}), \ldots \]

- stunning exception: \[ \Sigma^{++}_c (cuu) > \Sigma^0_c (cdd) > \Sigma^+_c (cud) \]

  \[ 2454.0 \pm 0.2 \quad 2453.8 \pm 0.2 \quad 2452.9 \pm 0.6 \text{ MeV} \]

- natural order restored for the bottom cousins: \[ \Sigma^-_b (bdd) > \Sigma^+_c (buu) \]

  \[ 5815.2 \pm 2.0 \quad 5807.8 \pm 2.7 \text{ MeV} \]

how do these patterns arise? heavy quark symmetry?
EFFECTIVE LAGRANGIAN at TREE LEVEL

• basic ingredients:
  
  Goldstone boson octet, symm. sextet and anti-symm. triplet in SU(3)

  \[
  \phi = \begin{pmatrix}
  \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
  \pi^- \\
  K^-
  \end{pmatrix} - \begin{pmatrix}
  \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
  \pi^+ \\
  K^0
  \end{pmatrix} - \begin{pmatrix}
  \frac{1}{\sqrt{6}} \eta \\
  \eta \\
  -\frac{2}{\sqrt{6}} \eta
  \end{pmatrix},
  \]

  \[
  B_{6c} = \frac{1}{\sqrt{2}} \begin{pmatrix}
  \sqrt{2} \Sigma_{c}^{++} \\
  \Sigma_{c}^{+} \\
  \Xi_{c}^{'+}
  \end{pmatrix} + \begin{pmatrix}
  \Sigma_{c}^{+} \\
  \sqrt{2} \Sigma_{c}^{0} \\
  \Xi_{c}^{0}
  \end{pmatrix} + \begin{pmatrix}
  \Xi_{c}^{'+} \\
  \Xi_{c}^{0} \\
  \sqrt{2} \Omega_{c}^{0}
  \end{pmatrix},
  \]

  \[
  B_{3c} = \begin{pmatrix}
  0 & \Lambda_{c}^+ & \Xi_{c}^+ \\
  -\Lambda_{c}^+ & 0 & \Xi_{c}^0 \\
  -\Xi_{c}^+ & -\Xi_{c}^0 & 0
  \end{pmatrix}.
  \]

• construction of the effective Lagrangian: non-linear \( SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \)

• isospin splittings through quark masses and virtual photons \( \rightarrow \) well behaved

• technology developed in the late 90ties at IKP-3 \( \text{Fettes, M., Müller, Steininger, \ldots} \)

• same for \textit{bottom} cousins \( (c \rightarrow b) \)
EFFECTIVE LAGRANGIAN at TREE LEVEL cont’d

- symmetry breaking terms at order $p^2$ as in the pion-nucleon case!

\[ \mathcal{L}_{\text{str.}}^{(2)} = -\langle \bar{B}_Q (\alpha_1 \chi_+ + \alpha_2 \langle \chi_+ \rangle) B_Q \rangle \]

\[ \mathcal{L}_{QQ}^{(2)} = -F_\pi^2 \langle \bar{B}_6 Q [\beta_0 (Q_+^2 - Q_-^2) + \beta_1 Q_+ \langle Q_+ \rangle + \beta_2 \langle Q_+^2 - Q_-^2 \rangle \\
+ \beta_3 \langle Q_+^2 + Q_-^2 \rangle] B_6 Q \rangle - F_\pi^2 \beta_4 \langle Q_+^T \bar{B}_6 Q + B_6 Q \rangle \]

- new symmetry breaking term at order $p^2$

\[ \mathcal{L}_{\text{em}}^{(2)} = \mathcal{L}_{QQ}^{(2)} - F_\pi^2 \beta_{1h} \langle \bar{B}_6 Q_+ \langle q_h \parallel \rangle B_6 Q \rangle \]

- physics behind this new term: static heavy quark charge $q_h$

\[ Q_B = 2Q + q_h \parallel = \begin{cases} 
  e \cdot \text{diag} \{2, 0, 0\}, & \text{for the charm baryons,} \\
  e \cdot \text{diag} \{1, -1, -1\}, & \text{for the bottom baryons,}
\end{cases} \]
LOWEST ORDER MASS SPLITTINGS

• mass splittings at order $p^2$

\[
\left( m_{\Sigma_c^+} - m_{\Sigma_c^0} \right)^{(2)} = 2\alpha_1 B_0 (m_u - m_d) + \frac{1}{6} F^2 \pi e^2 (\beta_0 - 2\beta_4 + 6\beta_{1h})
\]

\[
\left( m_{\Sigma_c^{++}} - m_{\Sigma_c^0} \right)^{(2)} = 4\alpha_1 B_0 (m_u - m_d) + \frac{1}{3} F^2 \pi e^2 (\beta_0 + \beta_4 + 6\beta_{1h})
\]

\[
\left( m_{\Xi_c^+} - m_{\Xi_c^0} \right)^{(2)} = 2\alpha_1 B_0 (m_u - m_d) + \frac{1}{6} F^2 \pi e^2 (\beta_0 - 2\beta_4 + 6\beta_{1h})
\]

\[
\Rightarrow \left( m_{\Xi_c^+} - m_{\Xi_c^0} \right) = \left( m_{\Sigma_c^+} - m_{\Sigma_c^0} \right) + \mathcal{O}(p^3)
\]

• experiment

\[-0.9 \pm 0.4 \quad - 2.3 \pm 4.2 \quad \text{MeV}\]

• good starting point, need to know the corrections
LEADING LOOP CORRECTIONS

- pion-baryon loops at order $p^3$
- no photon-baryon loops at order $p^3$
- resulting isospin splittings

\[ m_{\Sigma_c^+} - m_{\Sigma_c^0} = \Delta^{(2)}_{1c} + \Delta^{\text{loop}}_{1c}(m_{\Sigma_c}, m_{\Lambda_c}) + \mathcal{O}(p^4) \]

\[ m_{\Sigma_c^{++}} - m_{\Sigma_c^{0}} = \Delta^{(2)}_{2c} + \mathcal{O}(p^4) \]

\[ m_{\Xi_c'^+} - m_{\Xi_c'^0} = \Delta^{(2)}_{1c} + \mathcal{O}(p^4) \]

- loop function depends on 3 unknown LECs \( \rightarrow \) need three splittings as input
- symmetry breaking LECs come out of *natural size*
- extend to the *bottom* sector utilizing heavy quark symmetry (axial couplings)
PREDICTIONS and more

• We predict:

\[ m_{\Xi'_c} - m_{\Xi'_c}^0 = m_{\Sigma'_c} - m_{\Sigma_c}^0 - \Delta_{1c}^{\text{loop}} = -0.2 \pm 0.6 \text{ MeV} \ [ -2.3 \pm 4.2 ] \]

\[ m_{\Sigma_b^0} = \frac{1}{2} \left( m_{\Sigma_b^+} + m_{\Sigma_b^-} - \tilde{\beta}_4 \right) + \Delta_{1b}^{\text{loop}} = 5810.3 \pm 1.9 \text{ MeV} \]

\[ m_{\Xi'_b} - m_{\Xi'_b}^0 = \frac{1}{2} \left( m_{\Sigma_b^+} - m_{\Sigma_b^-} - \tilde{\beta}_4 \right) = -4.0 \pm 1.9 \text{ MeV} \]

• Explanation for the ordering:

The heavy-light photon exchange has a different sign for the charm and the bottom baryons. It is the interference of this term with the others that drives the behavior of the \( \Sigma_c \) iso-triplet. Also seen in D-meson splittings.
SUMMARY & OUTLOOK

• Big potential for precision physics at MAMI = important QCD tests
  → photoproduction ↔ pion-nucleon scattering lengths
  → precision determination of $a(\pi^+ n \rightarrow \pi^0 p)$
  → the $\pi^0 p$ scattering length is in reach
  → precision physics in $\eta$ decays
  → the cusp in $\eta' \rightarrow \eta\pi\pi$

• SU(3) chiral dynamics tests in kaon photoproduction
  → e.g. two-pole nature of the $\Lambda(1405)$

• Same methodology also useful in heavy-light systems

⇒ do not miss these great opportunities!