Two-Photon-Exchange Effects in Nucleon EM Form Factors

• Introduction
• Radiative Corrections
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• Future Experiments
• Summary

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MAMI & Beyond
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World Data Set on $G_E^p$ by mid 1990s

$Q^2 = 4EE'\sin^2(\frac{\theta}{2})$

$\varepsilon = \frac{1}{1 + 2(1 + \tau)\tan^2(\theta / 2)}$

$\sigma_R(Q^2, \varepsilon) = \varepsilon \left(1 + \frac{1}{\tau}\right) \frac{E}{E'} \frac{\sigma(E, \theta)}{\sigma_{Mott}} = (G_M^p)^2(Q^2) + \frac{\varepsilon}{\tau} (G_E^p)^2(Q^2)$
• John Arrington (PRC 68, 034325 (2003)) performed detailed reanalysis of SLAC data resulting in acceptable scatter of data
• Hall C Rosenbluth data (PRC 70, 015206 (2004)) in agreement with SLAC data
• No reason to doubt quality of either Rosenbluth or polarization transfer data
• Investigate possible theoretical sources for discrepancy

Large new data set based on polarization transfer showed linear decrease of $G_E^p/G_M^p$ with $Q^2$

In contrast with Rosenbluth data
Speculation: missing radiative corrections

Speculation: there are radiative corrections to Rosenbluth experiments that are important and are not included in the analysis.

Missing correction: linear in $\varepsilon$, not strongly $Q^2$-dependent.

$G_E$ term is proportionally smaller at large $Q^2$.

Effect more visible at large $Q^2$.

John Arrington
Radiative correction diagrams

Radiative corrections
\[ d\sigma = d\sigma_0 (1 + \delta_{RC}) \]

different in Super-Rosenbluth (proton detection)

Two-photon exchange box diagrams

Emission of soft real photons
Status of radiative corrections

- **Tsai (1961), Mo & Tsai (RMP 41, 205 (1969))**
  Box diagram calculated using only nucleon intermediate state and using $q_1 \sim 0$ or $q_2 \sim 0$ in both numerator and denominator (calculate 3-point function) $\rightarrow$ gives correct IR divergent terms

- **Maximon & Tjon (PRC 62, 054320 (2000))**
  Same as above, but make the above approximation only in numerator (calculate 4-point function)
  + use on-shell nucleon form factors in loop integral

- **Blunden, Melnitchouk, Tjon (PRC 72, 034612 (2005))**
  Further improvement by keeping the full numerator

- **Afanasev (hep-ph/0711.3065)**
  Including hard-photon emission in bremsstrahlung can account for $\sim$20% of discrepancy between Rosenbluth and polarization transfer

- **Bystritskiy, Kuraev, Tomasi-Gustaffson (PRC 75, 015207 (2007))**
  Claim that “leading and next-to-leading logarithmic approximation for hard-photon emission can explain full discrepancy”, but this can not be substantiated because of use of incorrect cut-off energies and implausible size of resulting radiative corrections
Guichon and Vanderhaeghen (PRL 91, 142303 (2003)) estimated the size of two-photon corrections (TPE) necessary to reconcile the Rosenbluth and polarization transfer data, introducing three generalized form factors $Y_{2\gamma}$, $\tilde{G}_E$ and $\tilde{G}_M$

$$\frac{d\sigma}{d\Omega} \propto \frac{\tilde{G}_M^2}{\tau} \left\{ \tau + \varepsilon \frac{\tilde{G}_E^2}{|\tilde{G}_M|^2} + 2\varepsilon \left( \tau + \frac{|\tilde{G}_E|}{|\tilde{G}_M|} \right) Y_{2\gamma}(\nu,Q^2) \right\}$$

$$\frac{P_t}{P_{t'}} \approx -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}} \left( \frac{|\tilde{G}_E|}{|\tilde{G}_M|} + \frac{1 - \frac{2\varepsilon}{1+\varepsilon}}{1+\frac{2\varepsilon}{1+\varepsilon}} \frac{|\tilde{G}_E|}{|\tilde{G}_M|} \right) Y_{2\gamma}(\nu,Q^2)$$

Need ~3% value for $Y_{2\gamma}$ (6% correction to $\varepsilon$-slope), independent of $Q^2$, which yields minor correction to polarization transfer, to bring the two data sets into agreement. However, these corrections are not supported by the $e^+/e^-$ cross-section ratio data.
Experimental Verification of TPE contributions

• Initially, the ratio of $e^+p$ and $e^-p$ cross section was the only available direct measurement of two-photon contributions.
• However, existing data were limited in statistics, $Q^2$-range and $\varepsilon$-range.
• Arrington (PRC 69, 032201 (2004)) demonstrated that TPE effects were non-zero with a 3σ-significance.
• He also extracted a TPE correction consistent with the $e^+/e^-$ ratio (PRC 71, 015202 (2005)).
Calculations of TPE effects

\[ \frac{d\sigma}{d\sigma_0} (1 + \delta) \]

\[ \delta = 2f(Q^2, \varepsilon) + \frac{2R\left\{M_0^\dagger M_1\right\}}{|M_0|^2} \]

\[ \delta_{2\gamma} = \frac{2R\left\{M_\gamma^\dagger M_{2\gamma}\right\}}{|M_\gamma|^2} \]

\( f(Q^2, \varepsilon) \) is the standard Mo & Tsai correction (soft photon exchange), which has some \( \varepsilon \)-dependence and is IR divergent.

IR divergent terms are canceled by soft-photon emission terms.

Two methods of calculating \( \delta_{2\gamma} \):

**Hadronic**
Use nucleon-pole diagrams with on-shell form factors in photon-nucleon vertices

**Partonic**
Factorize TPE amplitude into hard process of e-q scattering and a soft process described by GPDs
Hadronic TPE Calculations

• Calculation by Blunden, Melnitchouk, and Tjon (PRC 72, 034612 (2005)) resolves the discrepancy up to 2-3 GeV$^2$
  ➔ Only elastic contribution
  ➔ Linear $\epsilon$-approximation
• Note: TPE effect on cross section and polarizations are of same size
  ➔ Effect on $G_E$ amplified in high-$Q^2$ Rosenbluth measurements
  ➔ Most polarization measurements at large $\epsilon$, smaller TPE
TPE in Rosenbluth vs. Polarization Transfer

\[ \Delta (\varepsilon, Q^2) \]

- TPE corrections on \( \sigma \) as large as on \( P_T \) in polarization transfer
- Effect on Form Factor strongly enhanced by Rosenbluth separation

\[ Q^2 = 6 \text{ GeV}^2 \]

\[ P_L^{1+2Y} / P_L \]

\[ P_T^{1+2Y} / P_T \]

Mainz, March 31, 2009
In the BMT calculations the largest TPE contributions come from $\delta G_M$ (and at large $Q^2$ from $\delta G_E$)

Kondratyuk and Blunden (PRC 75, 038201 (2007)) have studied the nucleon resonance contributions, found large contribution from $\Delta$
Belushkin, Hammer and Meissner (PLB 658, 138 (2008)) performed separate dispersion analysis fits to EMFF data with only Rosenbluth and with only polarization transfer.

SC (Superconvergence) and pQCD denote the different constraints used for the dispersion relations.

Difference was then attributed to TPE and compared to BMT results.

**Solid:** $\Delta^{2Y}$ SC approach

**Dashed:** 1σ error bands

**Dotted:** $\Delta^{2Y}$ pQCD approach

**Dash-dotted:** BMT calculation
Effect on L-T Extractions

Arrington, Melnitchouk, Tjon
PRC 76, 035205 (2007)

full reanalysis of data, incorporating BMT calculations, but adding extra (small) phenomenological correction above $Q^2 = 1 \text{ GeV}^2$

\[ \delta_{2\gamma} = 0.01[\varepsilon - 1] \frac{\ln Q^2}{\ln 2.2} \]

\sim 1\% at 2 \text{ GeV}^2, 2\% at 5 \text{ GeV}^2

- Apply 100\% of the extra correction as an uncertainty (affects $G_M^p$ uncertainty)

- Corrections hardly visible in $e^+/e^-$ ratio
Partonic TPE Calculations

Afanasev et al. (PRC 93, 122301 (2004))

Model schematics:
- Assume factorization
- Hard eq-interaction
- GPDs describe quark emission/absorption
- Soft/hard separation

Polarization transfer
$1\gamma + 2\gamma$ (hard)
$1\gamma + 2\gamma$ (hard+soft)
E. Brash et al. (PRC 65, 051001 (2002)) have reanalyzed SLAC data with JLab $G_E^p/G_M^p$ results as constraint, using a similar fit function as Bosted. Reanalysis results in ~3% increase of $G_M^p$ data.

Arrington (PRC 71, 015202 (2005)) obtained similar results by implementing the TPE effects extracted from a comparison of Rosenbluth and polarization transfer data.
VEPP-3 positron-electron comparison

- Use VEPP-3 ring (50 mA)
- $e^+/e^-$ energy 1.6 GeV
- $\theta \approx 25^\circ$ and $65^\circ$
- $Q^2 \approx 0.3$ and $1.5 \text{ GeV}^2$
- Internal gas target
- Luminosity $\approx 5 \cdot 10^{31} \text{ (cm}^2\text{s})^{-1}$

J. Arrington, D. Nikolenko, spokespersons, nucl-ex/04-08020
VEPP-3 positron-electron comparison

Expected to run
Summer/fall 2009

Measurements at two $\varepsilon$-values
Projected slope based on TPE amplitudes from global extraction
$10.4 \pm 2.2 \%$ at $1.6 \text{ GeV}^2$
Positron-electron comparison in CLAS@JLab

- Use 5% radiator to generate photon beam, dump electron beam into tagger dump
- Put photon beam through 2% converter to generate positron/electron pairs
- Steer positron (electron) beam above (below) photon blocker with 4-dipole chicane system
- Overlapping positron and electron beams incident on hydrogen target
- Detect scattered lepton and recoiling proton in coincidence in CLAS - reconstruct initial lepton energy
Preliminary CLAS results

First engineering run successful

- $<Q^2> = 0.18 \text{ GeV}^2$ (CLAS)
- $Q^2 < 0.3 \text{ GeV}^2$ (SLAC)

Good agreement with SLAC data

Brian Raue (FIU) CLAS Collaboration Meeting, November 2008
- E05-007 Scheduled to run in late 2011
- Will provide $e^+/e^-$ cross-section ratios over large range of $Q^2$ and $\varepsilon$
pOsitron-proton
and
hYpothesis of
DoriS

Max. energy 4.45 GeV
Stored current 120 mA

eLectron-proton elastic scat
Multi-Photon exchange

OLYMPUS@DESY in DORIS
• Left-right symmetric
• Large acceptance:
  \[0.1 < Q^2/\text{GeV}^2 < 0.8\]
  \[20^\circ < \theta < 80^\circ, -15^\circ < \phi < 15^\circ\]
• COILS \[B_{\text{max}} = 3.8 \text{ kG}\]
• DRIFT CHAMBERS
  Tracking, PID (charge)
  \[\delta p/p = 3\%, \delta \theta = 0.5^\circ\]
• CERENKOV COUNTERS
  \(e/\pi\) separation
• SCINTILLATORS
  Trigger, ToF, PID (\(\pi/p\))
• NEUTRON COUNTERS
  Neutron tracking (ToF)

→ All the advantages of VEPP-3 expt.
  - Pure beam, well-defined energy
→ \((Q^2, \varepsilon)\) coverage close to CLAS
→ Coincidence measurement
  - Could do \((e,e'n)\), other reactions
Projected results for OLYMPUS

1000 hours each for \( e^+ \) and \( e^- \)
Lumi = \( 6 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1} \)

2008 - Full proposal submitted and approved
2009/10 - Transfer of BLAST
2011/12 - OLYMPUS Running
Benefits of improved LT separations

→ Compare precise LT and PT to constrain \textit{linear part of TPE} corrections

At large $Q^2$ values, $G_E \rightarrow 0$ and the $\varepsilon$-dependence is almost entirely from TPE

Extraction of TPE is almost as clean as in the $e^+/e^-$ comparison
Non-linearity Tests

\[ \sigma_{\text{red}} = P_0 \cdot [1 + P_1(\varepsilon - 0.5) + P_2(\varepsilon - 0.5)^2] \]

- Born approx \( \Rightarrow \sigma_R \) linear in \( \varepsilon \), TPE can have non-linearity
- SLAC NE11, JLab E01-001: quadratic terms consistent with zero
- Global fit, averaged over all \( Q^2 \) yields \( P_2 = 0.019 \pm 0.027 \)
- Project \( \delta P_2 \approx \pm 0.020 \) for both linearity scans, with global \( \delta P_2 \approx \pm 0.011 \)
- E05-017 will set meaningful limits over a wide \( Q^2 \)-range

Global linearity limits: V. Tvaskis et al., PRC 73, 025206 (2006)
E05-017: Return of “Super-Rosenbluth”

- E05-017: Ran in Hall C, Summer 2007
- Increase precision of LT separations; better measure of LT-PT difference
- Nearly all $\varepsilon$ dependence is TPE for $Q^2 > 5 \text{ GeV}^2$
- Precise linearity tests at $Q^2 = 0.983, 2.284 \text{ GeV}^2$

102 Kinematics points

$Q^2$ 0.40-5.76 $\text{GeV}^2$

13 points at $Q^2 = 0.983$
10 points at $Q^2 = 2.284$
• $\varepsilon$-Independent TPE amplitudes yield linear effect on cross section
• However, so do $G_M = 0$ and $Y_{2\gamma} = A + B/\varepsilon$
• Indistinguishable in $\sigma$, but very different in polarization transfer
• Experiment E04-019 measured polarization of recoil proton as function of $\varepsilon$, $\rho$ in Hall C late 2007
• GPD: Afanasev
• Hadronic: Blunden
• Predict different sign of TPE in PT
SSA in elastic eN scattering

- **Spin of beam OR target OR recoil proton NORMAL to scattering plane**

- On-shell intermediate state ($M_X = W$)

- Involves the imaginary part of two-photon exchange amplitudes

**Target:** general formula of order $e^2$
- GPD model allows connection of real and imaginary amplitudes
- Hadronic models sensitive to intermediate state contributions, no reliable theoretical calculations at present

**Beam:** general formula of order $m_e e^2$ (few ppm)
- Measured in PV experiments (longitudinally polarized electrons)
  - at SAMPLE, A4 (Mainz), $G^0$ and HAPPEX (JLab)
- Only non-zero result so far for TPEX

Peter Blunden
Available data on SSA

More recent calculations by Borisyuk & Kobushkin (arXiv:0812.0469)
**HAPPEX 2004 Hydrogen Transverse**

\[ E_e = 3 \text{ GeV}, \quad \theta_{\text{CM}} \sim 16^\circ, \quad Q^2 = 0.099 \text{ GeV}^2 \]

\[ A_T = -6.58 \text{ ppm} \pm 1.47 \text{ ppm (stat)} \pm 0.24 \text{ ppm (syst)} \]

**Total corrections \sim 200 \text{ ppb}**

**Dominant systematic errors:**
- Polarimetry (190 ppb)
- Beam asymmetry (100 ppb)
- Al background dilution (70 ppb, assumed \( A_T^\text{Al} = 0 \))
HAPPEX 2005 Helium Transverse

\[ E_e = 2.75 \text{ GeV}, \, \theta_{\text{lab}} \sim 6^\circ, \, Q^2 = 0.077 \text{ GeV}^2 \]

\[ A_T = -13.51 \text{ ppm} \pm 1.34 \text{ ppm (stat)} \pm 0.37 \text{ ppm (syst)} \]

**Dominant systematic errors:**
- Polarimetry (300 ppb)
- Linearity (140 ppb)
- Beam asymmetry (100 ppb)
- Al background dilution (120 ppb)

Normal beam asymmetry for elastic $e^-\text{^4He}$ scattering

Unitarity-based model predictions

**HAPPEX 2005 (preliminary)**

Without inelastic states, $10^{-9}$
TPE Beyond the Elastic Cross Section

• Important direct and indirect consequences on other experiments

  ➔ High-precision quasi-elastic experiments

  ➔ Proton charge radius, hyperfine splitting

  ➔ Strangeness from parity violation

  ➔ Neutron & Nuclear form factors

  ➔ Transition form factors

  ➔ Bethe-Heitler, Coulomb Distortion,...
General agreement that at least most of the discrepancy in the $G_E^p/G_M^p$ data determined by Rosenbluth separation and by recoil polarization transfer is due to two-photon exchange effects.

However, there is so far no quantitative proof of this.

Beam Single-Spin Asymmetry data have indeed proven the existence of TPE effects and data on the $e^+p/e^-p$ cross-section ratio indicate the existence of non-zero TPE effects.

Various theoretical models also support the resolution of the above discrepancy through TPE effects.

The three scheduled experiments that will provide accurate data on the $e^+p/e^-p$ cross-section ratio are anxiously awaited, but these will only cover a $Q^2$-range up to 3 GeV$^2$.

Thus, a continuous dedicated theoretical effort is required to complement such experimental studies.
THANK YOU!