Chapter 2

Experiments at MAMI and Theory

Theory

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Collaboration A1: Experiments with Virtual Photons

See sections H1 (page 76), H3 (page 153), and H5 (page 183).

Collaboration A2: Experiments with Real Photons

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Collaboration A4: Parity Violation

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Subproject H1: Structure of the Nucleon

H.-J. Arends / S. Scherer

H1.1 Introduction

The structure of the nucleon is a central topic of research at MAMI. Although the constituents of the nucleon, the quarks and gluons, cannot be directly observed, they are the basic agents that are responsible for the internal structure of the nucleon as put into evidence by:

- The spatial distribution of charges and magnetizations over a size of about $10^{-15} \text{ m}$, which can be measured by the 4 electromagnetic form factors.
- The spatial distribution of the axial current to be determined from the electroproduction of charged pions.
- The polarizabilities of the composite system under the influence of external electromagnetic fields as provided by real Compton scattering.
- The generalized polarizabilities that determine the spatial structure of the polarization densities to be measured by virtual Compton scattering.
- The existence of an excitation spectrum as described by pion cloud and resonance effects over the energy range of MAMI.
- The connections between energy-weighted integrals over the photoabsorption cross sections and ground state properties via dispersion relations and sum rules, which provide the possibility to test general principles of field theories and the internal consistency of Compton scattering and photoproduction.

The report on the experimental and theoretical activities of this project is organized as follows:

- Form factors of hadrons:
  - Electric and magnetic form factors of the nucleon
  - Axial form factor of the nucleon
  - Magnetic moments of nucleon resonances
- Polarizabilities of the nucleon:
  - Compton scattering and polarizabilities
  - Virtual Compton scattering and generalized polarizabilities
- Helicity structure of the nucleon and sum rules:
  - Helicity structure with real photons and sum rules
  - Structure functions of the nucleon and generalized sum rules
- Theoretical methods in effective field theory
H1.2 Electric and Magnetic Form Factors

H1.2.1 Measurement of the elastic electron-proton cross section and separation of the form factors \( G_e \) and \( G_m \) in the \( Q^2 \) region from \( 0.1 \text{(GeV/c)}^2 \) to \( 2 \text{(GeV/c)}^2 \)

J. C. Bernauer for the A1-Collaboration

New interest in the proton form factor has been stimulated by a precise measurement of the proton radius via Laser spectroscopy [1]. Their quite large rms-radius of 0.890 fm is in stark contrast to the old Stanford value of 0.805 fm [2]. With the old Mainz data [3, 4], there exists a very accurate measurement with the result 

\[
\begin{align*}
G_{e,p} \approx 0.857 \pm 0.008 \text{ fm} \quad \text{and} \quad G_{m,p} \approx 0.83 \pm 0.07 \text{ fm}
\end{align*}
\]

Recent \( G_{e,p} \)-measurements with polarization experiments [5, 6, 7], revealed at \( Q^2 \) beyond \( 1 \text{(GeV/c)}^2 \) drastic deviations from the dipole fit. New results for \( G_{e,n} \) (also from polarization measurements), together with the new situation for \( G_{e,p} \) led Friedrich and Walcher to have a closer look at the nucleon form factors [8]. They analyzed all four standard nuclear form factors \( (G_{e,p}, G_{e,n}, G_{m,p}, G_{m,n}) \) with the same phenomenological ansatz (eq. H1.1); the sum of a smooth part, \( G_s \) (eq. H1.2), described by two dipoles, and a bump, \( G_b \) (eq. H1.3):

\[
G_n = G_s + a_b Q^2 \cdot G_b
\]

\[
G_s = -\frac{a_{10}}{(1 + \frac{Q^2}{a_{10}})^2} + \frac{1 - a_{10}}{(1 + \frac{Q^2}{a_{21}})^2}
\]

\[
G_b = e^{-\frac{1}{2}\left(\frac{Q^2 - Q_b^2}{a_b}\right)^2} + e^{-\frac{1}{2}\left(\frac{Q^2 - Q_b^2}{a_b}\right)^2}
\]

This ansatz, which holds for all four form factors, reveals a bump/dip around \( Q^2 = 0.2 \text{(GeV/c)}^2 \), as shown in figure H1.1 for the proton form factors by the deviation of the existing data from the smooth part (eq. H1.2). A physical motivated ansatz led to comparable results.

![Figure H1.1: The difference between the measured form factors \( G_{e,p} \) and \( G_{m,p} \) and the smooth part (eq. H1.2) of the phenomenological ansatz.](image)

The existing data agree surprisingly well, but there exists not one continuous dataset over the whole bump/dip region. To establish the existence of this structure, a new experiment was conceived.
In [9], an exploratory measurement of the cross section of the reaction $\text{H}(e,e')p$ was done. The results show a rather large dependency on the position of the elastic line in the detector system. To diagnose this, the simulation package Simul++ has been extended to simulate the physical reactions in the detector system itself. Additionally, a technique to diagnose the position dependent single wire efficiencies of the VDC-system has been developed (see figure H1.2). To eliminate the problems found, the cathode foils of the VDC of spectrometer A have been replaced; non-working wires were repaired. A visual inspection of the old foils showed a strong relation between burned spots on the foils and areas with low efficiency as identified by aforementioned technique.

![Efficiency Map](image)

Figure H1.2: An example efficiency map generated by the new diagnosis technique. The red-to-black vertical lines are a result of non-working wires. The dark yellow spots identify areas where the foils were damaged.

The measured intersection points of a particle trace in the detector system and the physical properties of the particle in the target after the collision are connected by a complex relation. This relation is historically modelled by a polynomial expansion around the central trajectory; its coefficients were found experimentally by measuring elastic lines of heavy isotopes [11].

In the course of [9], a new method was developed. The relation was modelled by a spline ansatz which was fitted to pseudo elastic scattering data. A second spectrometer was used to tag the energy of the measured particle. This method needs less beamtime and leads to about 30% smaller missing mass peaks.

An experiment was proposed [10] and accepted by the PAC to measure the cross section of the reaction $\text{H}(e,e')p$ in the complete region accessible by MAMI and the 3-spectrometer setup. The planned kinematical setups are shown in figure H1.3. An analysis will be done not only using the standard Rosenbluth separation, but also with a direct fit of different form-factor ansätze to the measured cross sections. The targeted error-margin for each measured cross section is below 1%. Using simulated pseudo data, a global normalization to account for inefficiencies could be reproduced to the 0.3% level; the projected precision with these boundary conditions is shown in figure H1.4. First data for this experiment will be taken in the second half of 2006.
Figure H1.3: The kinematical setups of the proposed experiment in the $\varepsilon$-$Q$-space. Colored areas are excluded by the minimum/maximum incident beam energy (green), the minimum/maximum spectrometer angle (blue) and the maximum detectable scattered electron momenta (grey). The red line represents the maximum incident energy using only MAMI B. The dashed lines represent setups of constant angle (vertical) and constant incident energy (horizontal).

Figure H1.4: $G_e$ (left) and $G_m$ (right) of the phenomenological ansatz and anticipated errors from the proposed experiment (the thinner lines represent the estimated 1\(\sigma\)-errors). The errors are scaled by a factor of 100.
H1.2.2 Electromagnetic form factors of the nucleon in chiral perturbation theory including vector mesons

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The description of the electromagnetic form factors of the nucleon presents a stringent test for any theory or model of the strong interaction. Calculations in Lorentz-invariant baryon chiral perturbation theory up to fourth order fail to describe the proton and nucleon form factors for momentum transfers beyond $Q^2 \sim 0.1\text{GeV}^2$ [1, 2]. In Ref. [1] it was shown that the inclusion of vector mesons can result in the resummation of important higher order contributions. We calculate the electromagnetic form factors of the nucleon up to fourth order in manifestly Lorentz-invariant chiral perturbation theory with vector mesons as explicit degrees of freedom [3]. A systematic power counting for the renormalized diagrams is implemented using both the extended on-mass-shell renormalization scheme and the reformulated version of infrared regularization. We analyze the electric and magnetic Sachs form factors, $G_E$ and $G_M$, and compare our results with the existing data. The inclusion of vector mesons results in a considerably improved description of the form factors. We observe that the most dominant contributions come from tree-level diagrams, while loop corrections with internal vector meson lines are small.

H1.2.3 Unitary model for the $\gamma p \to \gamma \pi^0 p$ reaction and the magnetic dipole moment of the $\Delta^+ (1232)$

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Radiative pion photoproduction in the $\Delta (1232)$ resonance region is studied with the aim to access the $\Delta^+ (1232)$ magnetic dipole moment [1]. We present a unitary model of the $\gamma p \to \gamma \pi N$ ($\pi N = \pi^0 p$, $\pi^+ n$) reactions, where the $\pi N$ rescattering is included in an on-shell approximation. In this model, the low energy theorem which couples the $\gamma p \to \gamma \pi N$ process in the limit of a soft final photon to the $\gamma p \to \pi N$ process is exactly satisfied. We study the sensitivity of the $\gamma p \to \gamma \pi^0 p$ process at higher values of the final photon energy to the $\Delta^+ (1232)$ magnetic dipole moment. We compare our results with existing data and give predictions for forthcoming measurements of angular and energy distributions. It is found that the photon asymmetry and a helicity cross section are particularly sensitive to the $\Delta^+$ magnetic dipole moment.

The $\Delta (1232)$ is the first and most prominent excited state of the nucleon and the only well isolated nucleon resonance. Its properties provide an important test for theoretical descriptions in the non-perturbative domain of QCD. There are two kinds of electromagnetic properties of the $\Delta$. The first one involves the $N \to \Delta$ transition, described by the magnetic dipole ($\mu_{N\Delta}$) and electric quadrupole ($Q_{N\Delta}$) transition moments to be determined from pion electromagnetic production [2]. The other properties involve the $\Delta$ itself, the magnetic dipole moment $\mu_\Delta$, the electric quadrupole moment $Q_\Delta$, and the magnetic octupole moment of the resonance. In particular, the magnetic dipole moment (MDM) of the $\Delta (1232)$ is of considerable theoretical interest. In symmetric SU(6) quark models, the nucleon and the $\Delta$ resonance are degenerate and their magnetic moments are related through $\mu_\Delta = e_\Delta \mu_p$, where $e_\Delta$ is the electric charge of the $\Delta$, and $\mu_p$ the proton magnetic moment. However, different theoretical models predict considerable deviations from this SU(6) value [3]. The $\Delta (1232)$ MDM has also been investigated.
on the lattice at rather large quark masses [4], and very recently the chiral extrapolation of the Δ(1232) MDM, including the next-to-leading non-analytic variation with the quark mass, was also studied [5]. At present, there still is a considerably large gap in quark mass to bridge between the state-of-the-art lattice QCD calculations and the chiral limit. Therefore, it would be extremely helpful to know the resonance MDM for the physical quark mass values, through experiment. Unfortunately, the experimental information on the MDMs beyond the ground state baryon octet is very scarce. With the notable exception of the Ω− baryon, these higher nucleon resonances decay strongly, and thus have too short lifetimes to measure their MDMs in the conventional way through spin precession measurements.

The magnetic moment of the Δ⁺⁺(1232) has been measured by the reaction $\pi^+ p \to \gamma\pi^+ p$ [6]. As a result of these measurements, and using different theoretical analyses, the PDG [7] quotes the range $\mu_{\Delta^++} = 3.7 - 7.5 \mu_N$ (where $\mu_N$ is the nuclear magneton), while SU(6) symmetry results in the value $\mu_{\Delta^++} = 5.58 \mu_N$. The large uncertainty in the extraction of the experimental value is due to large non-resonant contributions to the $\pi^+ p \to \gamma\pi^+ p$ reaction because of bremsstrahlung from the charged pion ($\pi^+$) and proton (p). For the $\Delta^+(1232)$, it has been proposed [8] to determine its magnetic moment through measurement of the $\gamma p \to \gamma\pi^0 p$ reaction. A first measurement has only recently been reported by the A2/TAPS collaboration at MAMI [9]. At present, dedicated experiments are being performed with much higher count rates by using $4\pi$ detectors, such as the Crystal Ball detector at MAMI [10]. The analysis of this next generation of dedicated experiments requires a substantial theoretical effort aimed at minimizing the model dependencies in the extraction of the $\Delta^+$ MDM from the measurement of the $\gamma p \to \gamma\pi^0 p$ observables. First estimates for the reaction $\gamma p \to \gamma\pi^0 p$, including only the $\Delta$-resonant mechanism, were performed in Refs. [11, 12]. An improved calculation which contains both the $\Delta$-resonant mechanism and a background of nonresonant contributions has subsequently been carried out in Ref. [13]. This model was used in the analysis of the pioneering measurement of the $\gamma p \to \gamma\pi^0 p$ cross sections and an initial value of $\mu_{\Delta^+} = \left[2.7^{+1.0}_{-1.3} \text{(stat.)} \pm 1.5 \text{(syst.)} \pm 3 \text{(theor.)}\right] \mu_N$ has been extracted in Ref. [9]. Although the tree level model of Ref. [13] gives a qualitatively good description of the data of Ref. [9], a detailed quantitative comparison requires the inclusion of rescattering effects. It is therefore the aim of our present work to describe the radiative pion photoproduction by a properly unitarized theory.

In the soft photon limit ($E'_\gamma \to 0$), gauge invariance provides a model-independent relation between the cross sections for the $\gamma p \to \gamma\pi N$ and $\gamma p \to \pi N$ reactions,

$$R = \frac{1}{\sigma_\pi} \frac{d\sigma}{dE'_\gamma} \to 1 \quad \text{for } E'_\gamma \to 0$$  \hspace{1cm} (H1.4)

where $\sigma_\pi$ is an averaged cross section for the reaction $\gamma p \to \pi N$ and $d\sigma/dE'_\gamma$ the five-fold differential cross section for $\gamma p \to \gamma\pi N$ integrated over the outgoing photon and pion angles. One readily observes from Fig. H1.6 (left panel) that our theoretical calculation for the product $E'_\gamma \cdot d\sigma/dE'_\gamma$ approaches a constant and thus fulfills that low-energy theorem.

In the right panel of Fig. H1.6, we show the ratio $R$ constructed from our theoretical calculations of the $\gamma p \to \gamma\pi^0 p$ and $\gamma p \to \pi^0 p$ reactions, and compare with the data of Ref. [9], where $\sigma_\pi$ is evaluated from the $\gamma p \to \pi^0 p$ data using. The first data for the $\gamma p \to \gamma\pi^0 p$ process of Ref. [9] show a clear deviation from the soft-photon limit value $R = 1$ with increasing values of $E'_\gamma$. One sees from Fig. H1.6 that our unitary model gives a good overall description of the $E'_\gamma$
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dependence of the $\gamma p \to \gamma \pi^0 p$ reaction throughout the $\Delta$-region. Compared with the tree-level model developed in Ref. [13], our unitary model reduces the cross section at larger values of $E'_\gamma$ and thus provides an improved description of the data.

In Fig. H1.7 we investigate the sensitivity of the pion angular distribution at $E_{lab}^{\gamma} = 400$ MeV and $E_{cm}^{\gamma} = 100$ MeV with regard to the anomalous magnetic moment of the $\Delta^+ (1232)$, $\kappa_{\Delta^+}$. The upper part of Fig. H1.7 shows a considerable change in the angular distribution of the differential cross section when varying $\kappa_{\Delta^+}$ between 0 and 6. However it is also obvious that extracting a value of $\kappa_{\Delta^+}$ from a fit to the angular distribution is difficult. The reason is that the differential cross section first decreases when increasing $\kappa_{\Delta^+}$ from the value $\kappa_{\Delta^+} = 0$, reaches a minimum around a value $\kappa_{\Delta^+} = 3$, and increases subsequently when increasing $\kappa_{\Delta^+}$ further. This behavior is due to interference and evidently complicates an accurate extraction of $\kappa_{\Delta^+}$ from the differential cross section. However, we found that the photon asymmetry, for linearly polarized incident photons, decreases monotonically when increasing $\kappa_{\Delta^+}$, as is displayed in the lower part of Fig. H1.7. In particular, the photon asymmetry varies between $+0.35$ and $+0.15$ when varying $\kappa_{\Delta^+}$ from 0 to 6.
Figure H1.7: Top: the angular distribution of the emitted pions for the $\gamma p \rightarrow \gamma \pi^0 p$ 3-fold differential c.m. cross section $d\sigma/dE'\gamma d\Omega_\pi$ at incident photon lab energy $E'_{\gamma,\text{lab}} = 400$ MeV and fixed outgoing photon energy $E_{\gamma,\text{c.m.}} = 100$ MeV. The sensitivity of the unitary model to different values of $\kappa_{\Delta^+}$ is shown. Bottom: same for the photon asymmetry $\Sigma$.

Figure H1.8 shows the dependence of the helicity cross sections on the outgoing photon energy for the $\gamma p \rightarrow \gamma \pi^0 p$ reaction with a circularly polarized photon beam and a longitudinally polarized proton target, parallel ($\sigma_{3/2}$) or anti-parallel ($\sigma_{1/2}$) spins. In the low energy limit ($E'_{\gamma} \rightarrow 0$), one exactly recovers the helicity cross sections of the $\gamma p \rightarrow \pi^0 n$ reaction. At the higher values of $E'_{\gamma}$, one notices an interference pattern in the $\sigma_{3/2}$ cross section that strongly reduces the dependence on $\kappa_{\Delta^+}$ in the range from 0 to 3. The $\sigma_{1/2}$ cross section, on the other hand, decreases monotonically with increasing $\kappa_{\Delta^+}$, thus indicating a very strong sensitivity to the $\Delta^+$ MDM in the range 70 MeV $\leq E'_{\gamma} \leq 120$ MeV.

Besides a prediction for the $\gamma p \rightarrow \gamma \pi^0 p$ observables, our unitary model also provides a description of the $\gamma p \rightarrow \gamma \pi^+ n$ reaction. Since the $\gamma p \rightarrow \gamma \pi^+ n$ process is dominated by non-resonant processes and bremsstrahlung contributions originating from radiation off the charged pion, such an experiment will put stringent constraints on our theoretical description of the non-resonant contributions.


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Figure H1.8: The helicity dependence of the $\gamma p \rightarrow \gamma \pi^0 p$ c.m. cross section $d\sigma/dE/d\Omega_\pi$, divided by its soft photon value, as function of the outgoing photon energy $E_\gamma^{c.m.}$, at incident photon lab energy $E_{\gamma}^{lab} = 400$ MeV and pion emission angle $\theta_\pi^{c.m.} = 90^\circ$. Upper (lower) panel shows the cross sections for total helicity $3/2$ (1/2), respectively. The curves are the predictions of the unitary model for different values of $\kappa_\Delta$.


H1.3 Axial and Induced Pseudoscalar Form Factors

H1.3.1 Pion production in manifestly Lorentz-invariant baryon chiral perturbation theory

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Electromagnetic pion production is an important tool for investigating the internal structure of the nucleon. The aim of the current research is an analysis of both pion photoproduction and pion electroproduction in the framework of manifestly Lorentz-invariant baryon \( \chi \)PT. Previous calculations of pion production in \( \chi \)PT encountered the following difficulties: The early relativistic calculations suffered from the absence of a consistent chiral power counting [1]. On the other hand, calculations in the framework of heavy-baryon \( \chi \)PT involve approximations leading to a violation of crossing symmetry [2]. The infrared regularization [3] solves these problems. We calculate the invariant amplitude of electromagnetic pion production in the one-photon-exchange approximation up to fourth order. The result consists of nine tree-level diagrams and 85 one-loop diagrams. To achieve this goal we developed a program based on the existing package FeynCalc [4]. This program allows us to determine the gauge-invariant amplitudes and the subtraction terms of the amplitudes. The latter part is based on the development of a mathematical algorithm which enables us to calculate the subtraction terms of an integral directly in the framework of the infrared regularization. Hence we do not need to apply the Veltman-Passarino method. The next step is the numerical evaluation of the analytical results and the determination of the low-energy constants.


H1.4 Compton Scattering and Polarizabilities

H1.4.1 Status report on Compton scattering in baryon chiral perturbation theory to \( \mathcal{O}(q^4) \)

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The aim of this work is to calculate the doubly virtual, virtual, and real Compton scattering amplitudes in baryon chiral perturbation theory (BChPT) using the Lagrangian of Ref. [1] to order \( q^4 \), where \( q \) stands for a small quantity like the pion mass or small external momenta. From these amplitudes we can calculate physical observables such as cross sections or polarizabilities.

We have to calculate a total of 23 diagrams at tree order and 96 one-loop diagrams, where the crossed diagrams are taken into account as well. The loop diagrams are renormalized using the reformulated infrared renormalization scheme of Ref. [2]. For the numerical evaluation of the loop integrals we use the LoopTools library [3]. Due to the large number of diagrams we
make use of computer algebra systems, i.e. Form [4] and Mathematica. In order to calculate the diagrams in a reasonable time we have written procedures in Form and Mathematica to deal with the Dirac and isospin algebra, the subtraction terms needed for renormalization, and the extraction of the gauge-invariant amplitudes. We have checked these procedures either by hand or using existing Mathematica packages like FeynCalc [5]. So far all one-loop diagrams have been calculated. The next step will be to recalculate these amplitudes independently and to calculate the tree-order diagrams, enabling us to proceed with the extraction of physical quantities.


H1.5 Resonances in Effective Field Theory

H1.5.1 Universality of the rho-meson coupling in effective field theory

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In Ref. [1] we considered the effective Lagrangian of Ref. [2] describing the interaction among \( \rho \) mesons, pions, and nucleons. In principle, the Lagrangian contains all interaction terms which respect Lorentz invariance, the discrete symmetries, and chiral symmetry. As was stressed in Ref. [2], the equality of the \( \rho \pi \pi \) and the \( \rho NN \) coupling constants does not follow as a consequence of the symmetries of the Lagrangian. We performed a one-loop order analysis of the nucleon and \( \rho \)-meson self-energies as well as the \( \rho \rho \rho \) and \( \rho NN \) vertex functions. In accordance with the general principles of effective field theory, we required that all ultra-violet (UV) divergences can be absorbed into the redefinition of fields, masses, and coupling constants, as long as one includes every one of the infinite number of interactions allowed by symmetries. The renormalization procedure imposes consistency conditions among the (renormalized) parameters of the Lagrangian. In our case, both the universal coupling of the \( \rho \) meson as well as the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) value of the \( \rho \)-meson coupling constant [3, 4] turn out to be consequences of the self-consistency conditions imposed by the EFT approach.


H1.5.2 Quantum electrodynamics for vector mesons

D. DJUKANOVIC, M.R. SCHINDLER, J. GEGELIA, S. SCHERER

General principles of effective field theory [1] require that all ultraviolet-divergences (UV) can be absorbed into the redefinition of fields and parameters of the most general Lagrangian. Imposing the renormalizability in this sense one finds that not all parameters of the most general Lagrangian are free but satisfy consistency conditions. We used this argument in Ref. [2] and performed a one-loop order analysis of the $\rho$ meson self-energy and the $\rho\bar{\psi}\psi$ vertex functions. In order to get UV finite results we demanded that the sum over the divergent parts of the one-loop diagrams and the counterterm diagrams vanish. This way we obtained relations among the renormalized parameters leading to the fixed value $g = 2$ for the gyromagnetic ratio of the charged $\rho$ mesons. Furthermore the mass difference between the charged and neutral $\rho$ is given by

$$M_{\rho^0} - M_{\rho^\pm} \sim 1 \text{ MeV}$$

which has to be compared with the present PDG average of $(0.7 \pm 0.7) \text{ MeV}$ [3].


H1.5.3 Including the $\Delta(1232)$ in baryon chiral perturbation theory

C. HACKER, N. WIES, J. GEGELIA, S. SCHERER

Chiral perturbation theory (ChPT) has been very successful in describing the area of low-energy particle physics [1, 2, 3]. The $\Delta(1232)$ is the most prominent and well studied nucleon resonance and plays an important role in the phenomenological description of low- and medium-energy processes. This is due to the strong coupling of the $\Delta(1232)$ to the $\pi N$ channel and the relatively small mass difference between the nucleon and the $\Delta(1232)$.

In Ref. [4] we have considered the inclusion of the $\Delta(1232)$ as an explicit dynamical degree of freedom in manifestly Lorentz-invariant baryon chiral perturbation theory (BCtPT). The requirement of the consistency of the corresponding effective field theory in the sense of having the right number of degrees of freedom, leads to non-trivial constraints among coupling constants of various interaction terms. These constraints are compatible with the symmetries underlying the effective theory. Implementing them in the effective Lagrangian and using the extended on-mass-shell renormalization scheme [5] (or the reformulated version of the IR renormalization [6]) in combination with the higher-derivative formulation [7] we have obtained a consistent effective field theory with a systematic power counting.

Thus, we are in a position to calculate low-energy processes involving pions, nucleons, and deltas to any specified order in a small parameter expansion. As applications we have considered the $O(p^3)$ contributions to the nucleon mass, the pion-nucleon sigma term, and the pole-mass of the $\Delta(1232)$.

H1.5.4 Consistency of the \( \Delta \pi \) interaction in chiral perturbation theory

N. WIES, J. GEGELIA, S. SCHERER

Considering baryon chiral perturbation theory with explicit \( \Delta \) degrees of freedom (\( \Delta \)ChPT) one encounters the highly non-trivial problem of a consistent interaction of higher-spin fields. In a Lorentz-invariant formulation of a field theory involving particles of higher spin (\( s \geq 1 \)), one necessarily introduces unphysical degrees of freedom. Therefore, one has to impose constraints which specify the physical degrees of freedom. To write down interaction terms which lead to the correct number of physical degrees of freedom has proven to be a difficult problem. We analyze the constraint structure of a spin-3/2 particle interacting with a pseudoscalar (\( \Delta \pi \) interaction) using the Hamilton formalism.

There are already various suggestions for constructing consistent interactions involving spin-3/2 particles, but in this context we note that the problems showing up only for large field configurations are not relevant to low-energy effective field theories because these deal with small fluctuations of field variables around the vacuum. For larger field configurations the higher-order terms (infinite in number) generate contributions to physical quantities which are no longer suppressed by powers of small expansion parameters. Therefore, for large fluctuations the conclusions drawn from an analysis of a finite number of terms of the effective Lagrangian cannot be trusted. On the other hand, for small fluctuations around the vacuum one requires that the theory describes the right number of degrees of freedom in a self-consistent way. The interaction terms can be analyzed order by order in a small parameter expansion.

This analysis, discussed in Ref. [1], leads us to two constraints among the three lowest-order \( \pi \Delta \) interaction terms. From these constraints we find that the total Lagrangian is invariant under the so-called point transformation. On the other hand, demanding the invariance under the point transformation alone is less stringent and produces only classes of relations among the coupling constants.


H1.6 Helicity Structure of the Nucleon and Sum Rules

H1.6.1 Helicity Structure of the \( \gamma p \) interaction and GDH Sum Rule

The GDH-Collaboration has taken data on proton and deuterium targets at MAMI and ELSA between 1998 and 2003 using circularly polarized photons and longitudinally polarized nucleons provided by the Bonn frozen-spin target [1]. The photon induced reaction products were
registered by means of the large-acceptance detector DAPHNE [2]. A detailed description of the experimental setup can be found in [3] and references therein.

A summary of the present status for the proton is shown in Fig.H1.9, where the helicity difference $\Delta \sigma = \sigma_{3/2} - \sigma_{1/2}$ for the total cross section on the proton is compared to the unpolarized cross section.

As the large "helicity-blind" background of non-resonant photoproduction has almost disappeared in $\Delta \sigma$, one can expect the helicity difference to be a valuable observable to study the properties of nucleon resonances. Therefore, a detailed investigation of all partial reaction channels in the photon energy range of $200 \text{ MeV} < E_\gamma < 800 \text{ MeV}$ was carried out, which could be separated in the MAMI experiment due to the properties of the DAPHNE detector. During the last two years the analysis of single and double pion production data has been continued, most of the results have been published and will be briefly described.

H1.6.1.1 Single Pion Production in the $\Delta$-Region

In the low-energy region around the $\Delta(1232)$-resonance, the two single pion production channels give the dominant part of the GDH integral. Since the $\Delta$ has been studied extensively over many decades and a huge number of high quality data is available, one cannot expect to find any surprises here when looking at the double helicity observables. However, it can be taken as a confirmation, that the $E2/M1$ ratio of the $N \rightarrow \Delta$ transition obtained from the helicity observables (see Fig. H1.10) is fully consistent with the results from other methods.

The full data set from the GDH experiment in the $\Delta$-region is contained in Ref. [5].

H1.6.1.2 Single Pion Production in the $D_{13}(1520)$-Region

The single $\pi^0$-production, which has already previously been studied, has shown a high sensitivity to the $E2_-$ and $M2_-$ multipoles and therewith to the helicity amplitudes $A_{1/2}$ and $A_{3/2}$ of the $D_{13}(1520)$-resonance [6].
Figure H1.10: The helicity difference $\Delta \sigma = \sigma_{3/2} - \sigma_{1/2}$ for the reaction $\gamma p \rightarrow n\pi^+$ (Ref.[5]) in comparison with results from MAID. Solid curve: E2/M1=-2.5%, dashed curve: E2/M1 = 0, dotted curve: E2/M1=-5.0%.

Recently, the single $\pi^+$ production was studied in a similar way in this energy region. In Fig. H1.11 the different multipole contributions to $\sigma$ for the $n\pi^+$ channel as predicted by MAID (solution MAID03) and SAID (solution FA04K) analyses are shown. While both models give similar values for the total unpolarized $n\pi^+$ cross section (continuous lines of Fig. H1.11), the predicted behavior of the two main contributing multipoles ($E_{0+}$ and $E_{2-}$) are very different.

Figure H1.11: The most important multipole contributions to the total unpolarized total cross section for the $\gamma p \rightarrow n\pi^+$ reaction as predicted by MAID [7, 8] (upper figure) and SAID [9, 10] analyses (lower figure). Continuous line: sum of all multipoles; dash-dotted line: contribution due to the $E_{0+}$ multipole; dashed line: $M_{1+}$ multipole; dotted line: $E_{2-}$ multipole; continuous-dotted line: $M_{2-}$ multipole;

In the polarized case (see eq. H1.5) the negative sign of the $E_{0+}$ contribution and the presence of the interference term between the $E_{2-}$ and $M_{2-}$ multipoles highlights the differences between the two models. The helicity dependent total cross section $\Delta \sigma_{31} = (\sigma_{3/2} - \sigma_{1/2})$, where the
subscripts \((1/2)3/2\) correspond to the (anti)parallel \(\gamma\)-nucleon spin configuration, is given by:

\[
\Delta \sigma_{31} \propto |E_{0+}|^2 - |M_{1-}|^2 - 3|M_{1+}|^2 - 6E_{1+}^*M_{1+} - 3|M_{2-}|^2 + |E_{2-}|^2 + 6E_{2-}^*M_{2-} + \ldots
\]  

(H1.5)

In Fig. H1.12 we present examples of the obtained helicity dependent differential cross section \(\Delta \sigma_{31} = (d \sigma/d \Omega)_{3/2} - (d \sigma/d \Omega)_{1/2}\) for the \(n\pi^+\) channel from 643 MeV up to 783 MeV and compare these data to the previously mentioned models.

![Image of a graph showing the measured polarized differential cross section \(\Delta \sigma_{31}\) for \(\gamma p \rightarrow n\pi^+\) reaction for photon energies from 643 to 623 MeV (solid circles) compared to the model predictions. Curves as in Fig. H1.11. The errors shown are statistical only.](image_url)

Figure H1.12: The measured polarized differential cross section \(\Delta \sigma_{31} = (d \sigma/d \Omega)_{3/2} - (d \sigma/d \Omega)_{1/2}\) for the \(\gamma p \rightarrow n\pi^+\) reaction for photon energies from 643 to 623 MeV (solid circles) is compared to the model predictions. Curves as in Fig. H1.11. The errors shown are statistical only.

The total polarized cross section \(\Delta \sigma_{31} = (\sigma_{3/2} - \sigma_{1/2})\) was also evaluated from these data. The extrapolation into the unobserved region was made using the MAID calculation, which better reproduces the measured helicity dependent differential cross sections. In Fig. H1.13 the evaluated helicity dependent total cross section \(\Delta \sigma_{31} = (\sigma_{3/2} - \sigma_{1/2})\) is shown together with our previous polarized data at \(E_\gamma \leq 450\) MeV [5] and compared to the SAID and MAID analyses. In contrast to the unpolarized case, there is now a clear systematic discrepancy between the prediction of SAID and MAID at \(E_\gamma \gtrsim 450\) MeV, with our data being in general closer to the latter prediction.

The reason for the discrepancy between the models can be traced down to the different predicted contributions of the most relevant multipoles for the \(\gamma p \rightarrow n\pi^+\) channel, as shown in Fig. H1.11.

A deeper insight into the contribution of the different multipoles can be obtained from the separate evaluation of the two helicity dependent total cross sections \(\sigma_{1/2}\) and \(\sigma_{3/2}\) that can be performed from the combination of the measured unpolarized and polarized total cross sections. The results of this separation are shown in Figs. H1.14(a) and H1.14(b) for \(\sigma_{1/2}\) and \(\sigma_{3/2}\) respectively. In these figures, the model predictions from SAID and MAID are also shown together with the same observables evaluated from our previously published results at \(E_\gamma \leq 450\) MeV [5].
H1.6.2 Helicity Structure of the $\gamma d$ interaction and GDH Sum Rule

The Gerasimov-Drell-Hearn sum rule for real photons and any compound system with spin $S$ reads

$$\int_0^\infty \frac{\sigma_p(\omega) - \sigma_a(\omega)}{\omega} d\omega = \frac{4\pi^2e^2}{m^2} \kappa^2 S, \quad (H1.6)$$

connecting the ground state properties mass, $m$, and anomalous magnetic moment, $\kappa$, of the given system with the difference of the total photo-absorption cross sections for parallel ($\sigma_p(\omega)$) and antiparallel ($\sigma_a(\omega)$) alignment of the photon helicity and the system's spin. This relation is derived using very basic ingredients e.g. Lorentz and gauge invariance, unitarity, causality, and no-subtraction hypothesis [15].

An experimental test of the GDH sum rule requires doubly polarized experiments covering a very wide (theoretically infinite) energy range. Such experiments were carried out in 1998 and 2003 at the tagged photon facility in our institute. The experiments covered a photon lab energy range from pion production threshold to 800 MeV. The experimental setup was the same as for the proton runs, except for the target material. In both experiments, a polarized d-butanol target ($C_4H_9OH$ where all hydrogen atoms (H) were replaced with deuterium ($^2H$)) has been used which was provided by the Bochum and Bonn target groups [1]. The average target polarization for the 1998 pilot runs was approx. 30%. Due to a significant improvement in the target material it was possible to increase this polarization to values around 75% in the 2003 runs. This report will focus on the status of the analysis of these experiments.


H1.6.2.1 Data Analysis and Results

This section will give an overview of the status of the analyses and the results that have been produced so far. It is split in two parts.

1. Analysis and results for the 1998 pilot experiment: The analysis of this data set is more or less finished; several papers are soon to be published.

2. Status of the analysis of the 2003 data: This analysis is much less complete. The detector calibration is done and the major experiment parameters (photon flux, beam/target polarization) are determined. First extremely preliminary results indicate a nice agreement between unpolarized 1998 and 2003 data sets.

H1.6.2.2 \( \bar{\gamma}D \) pilot experiment

H1.6.2.3 Total Cross Section

An inclusive method was used to extract the total photo-absorption cross sections \( \sigma_p \) and \( \sigma_a \). This method has already been applied to data from the proton, both unpolarized and polarized [17], and to unpolarized deuterium data as well [19].

Since DAPHNE was optimized to detect charged particles, about 75% of the total photo-absorption cross section is accessible by the detection of events with charged hadrons in the final state. Approximately 15–20% of the total cross section can be found using events with one \( \pi^0 \) in the final state but no accompanying charged particle detected \( (N_{\pi^0}) \). The efficiency of DAPHNE for detecting \( \pi^0 \) final states \( (\epsilon_{\pi^0}) \) was determined via a GEANT simulation. \( \epsilon_{\pi^0} \) is non-zero for all angles and energies, hence no extrapolation is necessary in this case. Only corrections \((\approx 5\%)\) for charged pions emitted into angular and momentum regions outside DAPHNE’s acceptance \( (\Delta N(\pi^\pm)) \) are needed.

Using the above notation, the total photo-absorption cross section can be written as

\[
\sigma_{\text{tot}} \propto N_{\text{Hadrons}} = N_{\text{ch}} + N_{\pi^0} \epsilon_{\pi^0} + \Delta N(\pi^\pm). \tag{H1.7}
\]

Figure H1.15 shows results on the unpolarized deuteron as well as results of a former measurement also using DAPHNE [19]. The good agreement indicates that the detector is well understood and the analysis method can be applied to polarized data.

Results for the total cross section difference \((\sigma_p - \sigma_a)\) are shown as filled circles in Figure H1.16 [21] together with the latest results from the Bonn experiment at ELSA [13] between 815 and 1825 MeV (open circles). The data are confronted with calculations by Arenhövel’s group [22]–[24] (solid curve) and a MAID 2003 calculation (dotted curve) that just sums single pion contributions only for the free proton and the free neutron. The differences between the two model predictions and the data are seen much more clearly in Figure H1.17 which depicts the GDH integral function, also known as running GDH integral,

\[
I_{\text{GDH}}(E_\gamma) = \int_{200 \text{ MeV}}^{E_\gamma} \frac{\sigma_p(\omega) - \sigma_a(\omega)}{\omega} d\omega \tag{H1.8}
\]
Figure H1.15: Energy dependence of the unpolarized total photo-absorption cross section $\sigma_{\text{tot}}$ on the deuteron. Open circles: 1998 data [21]. Filled circles: results from a former measurement also using DAPHNE detector [19]. The systematic error is represented by the error band.

Figure H1.16: Results for the difference of the total photo-absorption cross sections for the two relative spin configurations $\sigma_p$ and $\sigma_a$ between 200 and 800 MeV photon energy. The 1998 pilot experiment data are represented by the filled circles. Between 815 MeV and 1825 MeV, results from the Bonn experiment [13] are also shown (open circles). The curves show theoretical calculations which are explained in more detail in the text.

Figure H1.17: Running GDH integral for the deuteron. For details see text.
in units of $\mu b$. The Arenhövel calculation agrees with the data far into the second resonance region, while it is clear that MAID fails because binding effects, deuteron disintegration, coherent $\pi^0$ production, double pion production and final state interactions are not taken into account.

### H1.6.2.4 Photodisintegration

The analysis for photodisintegration below $E_\gamma = 450$ MeV requires only events with one charged track in DAPHNE. Particle identification is achieved using an extended $\Delta E/E$-method, named “range fit,” given in [11], and kinematics of this two-body process allows for separation from competing reactions. Corrections for detection efficiency and for solid angle due to the finite target length were determined with GEANT simulations. Again, this analysis was first applied to unpolarized deuterium data giving results that nicely agree with data from [12].

Above $E_\gamma = 450$ MeV a considerable fraction of the protons from photodisintegration have enough kinetic energy to pass through DAPHNE and hence cannot be distinguished from pions originating from the reactions $\gamma + d \rightarrow N + N + \pi^\pm$. In addition, most of the single charged particle events in this energy region are due to pion photoproduction channels while protons from photodisintegration are only a small fraction of the total number of events. Due to these complications, the separation of the photodisintegration channel needs a more sophisticated analysis procedure that has not yet been applied. Therefore, only data for photon energies below 450 MeV will be presented here.

Results for differential photodisintegration cross sections for several photon energies are shown in Figure H1.18. Also shown are calculations by Schwamb et al. [22]–[25] as well as by Arenhövel [18]. The basic difference between both calculations is the retardation of the $NN$ potential and the meson-exchange that has been accounted for by Schwamb while retardation is not included in the Arenhövel model. Yet, only the calculations by Schwamb were used for the determination of the difference of the total photodisintegration cross sections to extrapolate to full solid angle coverage. The results are presented in Figure H1.19 together with the above mentioned calculations. The systematic error is 7.1% at 460 MeV photon energy.

### H1.6.2.5 Single Pion Production

The partial channels

\[ \gamma + d \rightarrow p_s + p + \pi^-, \quad \text{(H1.9)} \]
\[ \gamma + d \rightarrow n_s + n + \pi^+, \quad \text{(H1.10)} \]
\[ \gamma + d \rightarrow n_s + p + \pi^0 \quad \text{(H1.11)} \]

below 500 MeV photon energy have been taken care of as of yet. The index “$s$” denotes the nucleon being a spectator, i.e. the incoming photon interacts only with one nucleon in the deuteron while the second nucleon is emitted with its Fermi momentum. The spectator will not leave the target material which means it will not be detected. Hence, there is not enough information available to reconstruct the full kinematics of an event, i.e. only angles in the lab frame can be specified.
SUBPROJECT H1: STRUCTURE OF THE NUCLEON

Figure H1.18: Results for the dependence of the difference of differential photodisintegration cross sections on the polar angle $\theta$ of the proton for several photon energies $E_\gamma$ between 180 and 460 MeV [16]. $\theta$ is given for the centre-of-mass system in degrees. Only statistical errors are shown. The full line shows calculations by Schwamb et al. [22]–[25]. The dashed line represents calculations using the Bonn $r$-space potential as given in [18] additionally taking meson-exchange currents, isobar configurations and relativistic corrections into account.

Figure H1.19: Results for the energy dependence of the difference of the total photodisintegration cross sections $\sigma_p$ and $\sigma_a$ between 140 and 460 MeV photon energy [16]. Only statistical errors are shown. The lines are as specified in Figure H1.18.
Reaction (H1.9) can be uniquely accessed below $E_\gamma = 450$ MeV by selecting two charged tracks in DAPHNE, since all other possible partial channels result in at most one charged track in DAPHNE. Above $E_\gamma = 450$ MeV, one has to apply additional missing mass cuts to separate from $\gamma + \bar{d} \rightarrow p_s + p + \pi^- + \pi^0$.

The only way to access the Reactions (H1.10) and (H1.11) is the detection and identification of the charged pion and the proton respectively in the final state. This means that no reconstruction of the full kinematics on an event by event basis is possible.

Figure H1.20: Difference of the differential cross sections versus $\theta$ of the pion in the laboratory system for the reaction $\gamma + \bar{d} \rightarrow p_s + p + \pi^-$ (full circles). Also shown are calculations by A. Fix (full line) and from MAID for the free neutron reaction $\gamma n \rightarrow p\pi^-$ (dashed line). The figure was taken from [20]. For details see text.

Figure H1.20 depicts preliminary results for the difference of the differential cross sections $(d\sigma/d\Omega)_p - (d\sigma/d\Omega)_a$ versus $\theta$ of the pion in lab frame for the reaction (H1.9). The full curve shows calculations by A. Fix [14], the dashed curve gives the contribution of $\pi^-$ production on the free neutron.

### H1.6.2.6 Double Pion Production

The only channel that has been analyzed yet is

$$\gamma + d \rightarrow p + p + \pi^- + \pi^0.$$
Since a significant part of the detected particles is emitted in forward direction, not only DAPHNE is used in the analysis but also MIDAS, a microstrip silicon detector covering polar angles between 7.5 and 16.5 degrees. For this, the analysis is basically split in two parts. Firstly, one asks for two charged tracks in DAPHNE one of which is identified as a proton and the other as a pion. In addition, one photon from the decay of the \( \pi^0 \) is required in coincidence in DAPHNE. Alternatively, one charged track is required in DAPHNE and in MIDAS which are identified as proton and pion, respectively, together with a photon in DAPHNE. The contributions of competing reactions could be subtracted using GEANT simulations. Since the 1998 data provide only very limited statistics for this channel, also the 2003 data are used. Very preliminary results on unpolarized liquid deuterium data show nice agreement between 1998 and 2003 measurements.

H1.6.2.7 Experiment of 2003

Some basic work has been finished on the analysis of the 2003 data.

1. The detector system DAPHNE has been energy calibrated. This has been done as follows:

   (a) Using cosmic radiation, one determines the corrections for the mechanical imperfections of the wire chambers.

   (b) One can get the reaction kinematics of the charged particles from the reactions \( \gamma d \rightarrow p + n \) and \( \gamma p \rightarrow n + \pi^+ \) using the wire chamber information. An identification of these reactions and a separation from background at this stage of the calibration process is possible using \( \Delta E/E \)-plots with very generous cuts applied.

   (c) Since the geometry of the detector is well known, one can calculate the energy deposited by the respective particle from the reaction kinematics and plot this value versus the ADC-information of the according scintillator. For the bars that are read out from both sides, one can use the quantity \( \sqrt{\text{ADC}_1 \cdot \text{ADC}_2} \), where the position dependence of the ADC-values cancels in first approximation. This correction is necessary for A layer which is read out from one side only.

   (d) The resulting 2-dimensional plots, Figure H1.21, show a linear distribution of the registered events. A straight line fit to this distribution will give the energy calibration information.

2. The photon flux information for the normalization of the cross sections has been determined.

3. The time evolution of the electron beam polarization could not be quantified precisely yet due to a failure in the spin alignment settings of the accelerator in the July 2003 runs. So far, the data are analyzed assuming 75% electron polarization with an error of 5%.

4. The target polarization has been found to be in the order of 70%, with a relaxation time of about 200 h.

The analysis of the 2003 data for the channels mentioned in the 1998 experiment is currently in progress and results with lower statistical errors than shown before are soon to be expected.
Figure H1.21: Calibration of the DAPHNE detector.

\[ \gamma + p \rightarrow n + \pi^+ \]

\[ \text{ADC}_{\text{korr}} = 5.6 \cdot dE - 14.21 \]

\[ \gamma + d \rightarrow p + n \]

\[ \text{ADC}_{\text{korr}} = 5.58 \cdot dE - 15.36 \]


H1.6.3  Higher moments of nucleon spin structure functions in heavy baryon chiral perturbation theory and in a resonance model

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The forward scattering of spacelike virtual photons (with virtuality $Q^2$) on the nucleon, allows one to study sum rules which relate nucleon structure quantities to inclusive electroproduction cross sections. At large $Q^2$ such sum rules are obtained in deep-inelastic scattering. The study of these sum rules as function of $Q^2$ from the real photon point to large $Q^2$, opens up the perspective to interpolate between the non-perturbative and perturbative regimes of QCD, as one goes from low to high $Q^2$. In particular, the moment $d_2$ of the nucleon spin structure functions can be measured by scattering longitudinally polarized electron beams off nucleon targets with transverse and longitudinal polarizations. Being a higher moment in the Bjorken variable $x$, $d_2$ contains appreciable contributions from the resonance region. In Ref. [1] we have therefore studied the threshold and resonance contributions to $d_2$ within the frameworks of heavy baryon chiral perturbation theory (HBChPT) and of a unitary isobar model (MAID). Since a measurement of $d_2$ requires also transverse polarization, experimental information on this observable has become available only recently at SLAC [2, 3] and JLab [4]. Further experiments are underway or proposed at JLab. In particular, the SLAC experiments [3] yielded the values: $d_p^2 = 0.0032 \pm 0.0017$ and $d_n^2 = 0.0079 \pm 0.0048$ at $Q^2 = 5 \text{ GeV}^2$.

For the first moment $\Gamma_1^{(1)}$ of $g_1$ the twist expansion can be written as [5, 6]:

$$\Gamma_1^{(1)}(Q^2) = \Gamma_{1,tw-2}^{(1)}(Q^2) + \frac{M_N^2}{Q^2} (a_2(Q^2) + 4d_2(Q^2) + 4f_2(Q^2)) + O\left(\frac{M_N^4}{Q^4}\right).$$  \hspace{1cm} (H1.12)

The term proportional to $a_2(Q^2)$ arises due to target mass corrections and is given by the twist-2 part of the third moment of $g_1$, whereas the terms proportional to $d_2$ and $f_2$ in Eq. (H1.12) correspond to dynamical higher-twist corrections.

The term $d_2$ can be splitted into an elastic contribution,

$$d_2^{el}(Q^2) = \left(\frac{G_E(Q^2) + G_E(Q^2) - G_M(Q^2)}{2(1 + 4M_N^2/Q^2)}\right) G_M(Q^2),$$  \hspace{1cm} (H1.13)

which vanishes like $Q^{-8}$ for $Q^2 \to \infty$, and an inelastic contribution,

$$d_2^{inel}(Q^2) = \int_0^\infty dx x^2 \left(3 g_2(x, Q^2) + 2 g_1(x, Q^2)\right).$$  \hspace{1cm} (H1.14)

To evaluate Eq. (H1.14), we can equivalently express the third moment of the twist-3 part of $g_2$ in terms of the spin-dependent doubly virtual Compton scattering amplitude in the forward direction (VVCS). Following the notations of Ref. [7], we use the VVCS amplitudes $g_{TT}(v, Q^2)$...
and $g_{LT}(v, Q^2)$, where $v$ is the lab energy and $Q^2$ the virtuality of the virtual photon, and $T$ ($L$) denotes the transverse (longitudinal) virtual photon polarization.

The non-pole contributions to the real parts of these functions obey the following low energy expansion (LEX):

\[
\text{Re } g_{TT}(v, Q^2) - \text{Re } g_{TT}^{\text{pole}}(v, Q^2) = \left( \frac{2\alpha_{em}}{M_N^2} \right) I_4(Q^2) v + \gamma_0(Q^2) v^3 + o(v^3), \quad (H1.15)
\]

\[
\text{Re } g_{LT}(v, Q^2) - \text{Re } g_{LT}^{\text{pole}}(v, Q^2) = \left( \frac{2\alpha_{em}}{M_N^2} \right) Q I_5(Q^2) + Q \delta_{LT}(Q^2) v^2 + o(v^4). \quad (H1.16)
\]

The function $I_4(Q^2)$ in Eq. (H1.15) is a generalization of the GDH sum rule, and one recovers the GDH sum rule as $I_4(0) = -\kappa_N^2/4$, with $\kappa_N$ the nucleon anomalous magnetic moment. The higher order terms in Eqs. (H1.15) and (H1.16) can be expressed in terms of nucleon spin polarizabilities [7]. We find that the inelastic contribution to the third moment $d_2$ of Eq. (H1.14) can be expressed as

\[
d_{2}^{\text{inel}}(Q^2) = \frac{Q^6}{8M_N^2} \left\{ \frac{1}{M_N^3 Q^2} (I_4(Q^2) - I_1(Q^2)) + \frac{1}{2\alpha_{em} Q^2} \delta_{LT}(Q^2) \right\}, \quad (H1.17)
\]

where $I_1(Q^2)$ in the inelastic contribution to the first moment of the structure function $g_1$.

We have evaluated the integrals $I_4(Q^2)$ and $I_1(Q^2)$ as well as the longitudinal-transverse generalized forward spin polarizability $\delta_{LT}(Q^2)$ in HBChPT at $o(p^4)$. By combining these results we can now construct $d_{2}^{\text{inel}}$ from Eq. (H1.17) at $o(p^4)$.

In Fig. H1.22, we show the results for the $Q^2$ dependence of the moment $d_2$. It rises strongly at the lower $Q^2$ like $Q^6$, and tends to a small constant value asymptotically, corresponding with the twist-3 matrix element entering in the OPE of Eq. (H1.12). One sees from Fig. H1.22 that also the ChPT results rise strongly with $Q^2$. The large difference between the $O(p^3)$ and $O(p^4)$ results originates from the known large difference between the $O(p^3)$ and $O(p^4)$ HBChPT results for the forward spin polarizability $\gamma_0$ [8]. It is interesting to note however that the $O(p^3)$ HBChPT result is in good agreement with the phenomenological MAID estimate up to $Q^2 \approx 0.25$ GeV$^2$. The convergence of the heavy baryon chiral expansion for the forward spin polarizability has been recently investigated in Ref. [9] using the Lorentz invariant formulation of baryon ChPT of Ref. [10]. It was found that the main reason for the slow convergence of $\gamma_0$ in the $1/M_N$ expansion is due to the slow convergence of the Born graphs. However the relativistic result to fourth order in the chiral expansion, even when supplemented with $\Lambda$ and vector meson contributions, still does not agree with the data, which suggests yet higher order contributions.

Very recently, the inelastic contribution to $d_2$ has been measured for the neutron at intermediate $Q^2$ values at JLab/Hall A [4]. It is seen from Fig. H1.22 that the prediction of the MAID model shows an excellent agreement with these data. In particular, the MAID model predicts $\gamma_0 = -0.707 \cdot 10^{-4}$ fm$^4$ at the real photon point, which is in relative good agreement with the experimental value [7] $\gamma_0 = [-1.01 \pm 0.08 \text{ (stat)} \pm 0.10 \text{ (syst)}] \cdot 10^{-4}$ fm$^4$.

Furthermore, the quantity $f_2(Q^2)$ can be evaluated phenomenologically. From a comparison to the new JLab data we extract $f_2$ for $Q^2$ in the range $0.5 - 2$ GeV$^2$ as $f_2^2 \approx 0.15 \rightarrow 0.18$ and $f_2^2 \approx -0.026 \rightarrow -0.013$. 
Figure H1.22: $Q^2$ dependence of the moment $d_2$ for proton (upper panel) and neutron (lower panel). The dashed-dotted curve is the elastic contribution to $d_2$ according to Eq. (H1.13). The other curves represent the inelastic contributions to $d_2$. Solid curves: MAID estimate for the $\pi$ channel; dotted curves: $O(p^3)$ HBChPT; thick (upper) dashed curves: $O(p^3) + O(p^4)$ HBChPT; thin (lower) dashed curves: $O(p^3)$ HBChPT with $O(\epsilon^3)$ $\Delta$ contribution added. The JLab/Hall A data (diamonds) are from Ref. [4] (inner error bars are statistical errors only, outer error bars include systematical errors). The SLAC data (circles at $Q^2 = 5$ GeV$^2$) are from Ref. [3].

We can compare these phenomenological values for $f_2$ with several model estimates. In Ref. [11], an estimate was given within QCD sum rules which yielded as values: $f_2^p = -0.037 \pm 0.006$ and $f_2^n = -0.013 \pm 0.006$. In Ref. [12], the instanton vacuum picture was used to estimate matrix elements of quark gluon operators, with the results $f_2^p = -0.046$ and $f_2^n = +0.038$.

It has been suggested that the twist-3 moment $d_2$ and the twist-four moment $f_2$ are related to the response of the color electric ($\chi_E$) and magnetic ($\chi_B$) fields to the polarization of the nucleon in its rest frame [13], defined as:

$$\langle PS | \bar{\psi} \gamma^\mu \gamma^5 \gamma^\nu B \psi | PS \rangle = \chi_B 2 M_N^2 \vec{S},$$
$$\langle PS | \bar{\psi} \vec{\alpha} \times \vec{E} \psi | PS \rangle = \chi_E 2 M_N^2 \vec{S},$$

(H1.18)

where $P$ is the nucleon momentum and $S$ the projection of its spin vector $\vec{S}$. Furthermore $\vec{E}$ ($\vec{B}$) are the color electric (magnetic) fields respectively, and $g$ is the strong coupling constant.

With these definitions, the moments $d_2$ and $f_2$ can be expressed in terms of the gluon-field polarizabilities as,

$$d_2 = \frac{1}{4} (\chi_E + 2 \chi_B),$$
$$f_2 = (\chi_E - \chi_B).$$

(H1.19)

Since the experimental value of $d_2$ is of order $10^{-3}$, we see that the predicted central values of $f_2$ are larger than those of $d_2$ by about a factor 50 for the proton. These findings agree, at least
qualitatively, with estimates using QCD sum rules [11] and based on the instanton vacuum approach [12], but less so with the predictions of bag models [6]. Within the large uncertainties of all existing predictions, we may therefore conclude that $d_2 \ll f_2$. This observation can be combined with Eq. (H1.19) to yield $\chi_E \approx +\frac{2}{3} f_2$ and $\chi_B \approx -\frac{1}{3} f_2$. In particular a positive value of $f_2$ as found in the phenomenological extractions for the proton, leads to a negative value of $\chi_B$, i.e., color diamagnetism.


H1.7 Technical Developments for Helicity Structure Measurements

H1.7.1 Development of a frozen spin target for the crystal ball detector

In the physics program for he MAMI C accelerator the investigation of the spin structure of the nucleon is an important aim. This will be attacked in the framework of the A2-collaboration by doing double polarized experiments, using the linearly or circularly polarized energy labelled photon beam in combination with a new solid state polarized target. The main detector system will be the Crystal Ball, consisting of 672 NaJ Crystals in combination with the TAPS detector for the forward angles, leading to an angular acceptance close to $4\pi$. For the period of the measurements with the higher energy of new MAMI C accelerator stage (1.5GeV) it is foreseen to set up a new frozen spin target, which has to keep the high angular acceptance of the detector.
system. The main concept of this target is similar to that one of the Bonn frozen spin target [1], which was used in 1998 and 2003 for the measurement of the GDH sum rule on the proton and neutron in Mainz.

The operation of a solid state polarized target requires different, technical complicated subsystems, sketched in the picture H1.23. The knowledge of different polarized target groups around the world based on a more than 40 years history in developing polarized solid targets is used for realising the complex apparatus.

Central part of the target apparatus is a $^3$He—$^4$He dilution refrigerator. In contrast to electron beams the secondary beam of the Mainz A2 real photon facility causes not too serious beam heating problems for the target material, also radiation damage is no problem in our typical experimental conditions. The target cryostat needs to reach a base temperature of 50 mKelvin to provide in the condition of the frozen spin mode a long relaxation time for the target material. It is planned to build a horizontal cryostat with separator and evaporator in the pre-cooling stages.

The design has to be $\phi$-symmetrical to keep the detector geometry, the beam axis will by equal to the cryostat axis. The target material has to be loaded along the beam axis using a specially adopted target insert. This target insert needs to seal the cavity against the beam pipe vacuum. In addition a superconducting holding coil has to be integrated into the cryostat, operating at a temperature of 1 Kelvin. This coil has to be as thin as possible to keep the detecting thresholds for the outgoing particles as low as possible. A first version has been produced in the Mainz mechanics workshop in the year 2005. The beam height in the A2 tagger hall is 1760mm. The beam will pass along the axis of the dilution refrigerator. The Crystal Ball detector is complemented by an inner tracking and veto detector (wire chambers and plastic scintillators). This fixes the outer diameter of the forward target cryostat tube to a maximum of 93mm; the
opening angle of the ball in combination with the cabling of the inner detectors determines the length of the different cryostat parts.

Figure H1.24: The new dilution refrigerator for the Crystal Ball detector.

There has been formed a collaboration with the Dubna/Moskau Polarized Target Group with the goal of the design, construction and complete test of the new 3He/4He-Dilution cryostat. The design work has been finished in the year 2004 (see figure H1.24), the realization is on the way.

The polarizing magnet was already bought from the company Cryo-Technics. The solenoid coils are cooled in a liquid He bath (4 K); the outer thermal radiation shield is cooled by liquid nitrogen to minimize the helium consumption. The field axis is coincident with the beam axis, the maximum strength of the field is 2.5 T, and the uniformity is better than $\Delta B/B < 10^{-4}$ over the target volume of $6 \text{ cm}^3$. The warm bore of this magnet is 100mm to allow the entrance of the target refrigerator. The magnet has been successfully used by the group of Prof. Backe in Mainz is available now for the experiments with polarized target.

The Roots blowers with maximum pumping speed of $4000 \text{ m}^3/\text{h}$ were specially designed by the German company Pfeiffer to provide a very low leak rate ($< 10^{-5} \text{mBar} \cdot \text{l/sec}$), which is very important to pump the expensive 3He gas of the dilution refrigerator. The tuning of the different pumping stages are done that way, that it is possible to avoid an additional oil-sealed rotary pump in the main pumping circuit. This will provide a long operation time of the cryostat and avoid blockage by dirt in the restrictions of the cold part of the 3He circuit. The automatisation of the complete vacuum apparatus has been finalized in the year 2005. The pumping system consists of a series of pumps as follows: $4000 \text{ m}^3/\text{h}$, $2000 \text{ m}^3/\text{h}$, $1000 \text{ m}^3/\text{h}$, $500 \text{ m}^3/\text{h}$, $250 \text{ m}^3/\text{h}$ with two motors. Water-cooled heat exchangers are added to dissipate the heat of the compressed helium gas.
In the framework of a diploma thesis a microwave apparatus was developed and successfully tested in the GDH experiment in 2003. Special features of this computer-controlled apparatus with a center frequency of 70 GHz are a tunability of frequency of 300 MHz and a stability of better than 1 MHz. In addition a motor driven attenuator can adopt the microwave power to the requirements of the target. An additional frequency generator (IMPATT diode at 70 GHz) has been bought and tested in 2006.

We plan to use an NMR apparatus similar to the design of the Bochum polarized target group to measure the degree of target polarization. The NMR System is a serial resonance circuit with a coil within or around the target material. The coil is connected to the capacitor via a transition cable. A change in the polarization value induces a change in the susceptibility of the coil. This causes a variation of the Q factor of the circuit, which can be measured as an increase or a decrease of the voltage. This voltage is conditioned with rf- and lf- amplifiers. The signal is obtained by a frequency scan (sweep) over the resonance (Larmor) frequency. A PC with a Windows OS detects the prepared signal. In order to increase the signal to noise ratio, many signals are accumulated (the noise reduces by the square root of the number of sweeps). Collaborators from the Rudjer Boskovic Institute in Zagreb, Croatia have started to setup the NMR system in Mainz in the year 2005.


H1.7.2  Prestudy for the use of the polarized $^3$He target at the photon beam of MAMI

Polarized $^3$He targets play an important role in fundamental physics experiments because the nucleus of a polarized $^3$He atom consists of two spin paired protons and a single unpaired neutron, making it appear approximately as a single polarized neutron. Hence, the absence of free neutron targets make $^3$He a valuable tool in the study of the fundamental structure of the neutron.

An polarized target requires a magnetic holding field, $B_0$, to provide a quantization axis along which the nuclei are polarized. Therefore, one of the topics in the study for the use of this polarized $^3$He target is to produce and test a solenoid to provide this magnetic holding field.

On the other hand, the feasibility of a future research to be carried out with a polarized $^3$He gas target was investigated in a scattering experiment performed with the 855 MeV polarized electron beam at the MAMI accelerator. Circularly polarized photons were obtained by Bremsstrahlung of the longitudinally polarized electrons coming from the accelerator. A quartz glass cell with two kapton windows was inserted into the Crystal Ball detector and filled with $^4$He gas at typical working conditions. Because of $^4$He gas has similar properties as $^3$He gas, but is cheaper, $^4$He was chosen to perform this test. The results from the $^4$He test are comparable to those with $^3$He when applying a simple scaling factor.

The analysis of the data obtained with this measurements focused on identifying hadronic events from the particles scattered in the target cell and allowed the calculation of the ratio between the nuclear scattering events produced on the target cell windows and those produced on the $^4$He gas. The predictions for this ratio were calculated and are displayed in Fig.1.

This figure presents the normalized counts for the conditions imposed with the different cuts applied to subtract the background particles. Fig.1 shows the number of events for two different
measurements corresponding to empty target and the target filled with $^4$He gas at a pressure of 7 bar as a function of the position along the photon beam axis, measured by tracking using the MWPC. One can see three physical regions of significant contribution to the total number of events to be considered: the region corresponding to the air outside the target cell, the peak due to the entrance Kapton window and finally the target cell region. The measured value of 0.93 for the ratio is in good agreement with the expected value of 0.89.

Therefore, this positive result gives confidence in the performance of a future investigation with a polarized $^3$He gas target used in combination with the Crystal Ball detector.
H1.8 Theoretical Methods in Effective Field Theory

H1.8.1 Baryon masses in manifestly Lorentz-invariant baryon $\chi$PT

B.C. Lehnhart, J. Gegelia and S. Scherer

A determination of the baryon masses of the baryon octet is a test for $\chi$PT. The reasons for this project: The chiral perturbation theory has been really successful in describing the strong interactions of the pseudoscalar meson octet in the low energy region. But including the baryon sector problems arose: The nucleon mass in the chiral limit $M_0$ appears as a new mass scale and using a power counting analogous to the one in the mesonic sector means to give up manifest Lorentz invariance. The consequences are an increasing complexity of the Lagrangian and not all scattering amplitudes (HBChPT) show the correct analytical behaviour. The used extended-on-mass-shell (EOMS) renormalization scheme [1] allows a strict manifestly Lorentz invariant calculation. Using this renormalization scheme we have computed the baryon masses and the nucleon sigma term in a full calculation up to the third order and in a incomplete one up to the fourth order in the framework of manifestly Lorentz invariant baryon $\chi$PT. The fourth order is incomplete because, at this stage, we do not have sufficient information to constrain the full set of parameters contributing at fourth order. The fourth order contributions point towards a slow convergence, i.e. due to the relatively large mass of the strange quark, the convergence in the three-flavour sector is slower in comparison with the two-flavour sector. We performed a least squares fit for the parameters $M_0, b_0, b_D$ and $b_F$ using the empirical masses and three different values of the pion-nucleon sigma term as input. The other way round we determined the baryon masses for different symmetries of the lagrangian (fig. H1.26). We compared our results with other calculations in the framework of $\chi$PT. At the third order, our expressions (EOMS) for the masses and the sigma-terms are the same as in the heavy-baryon formulation [2] but differ from the infra-red formulation [3]. A consistent consideration of the baryon decuplet seems to be important in the $SU(3)$ sector [4].

![Mass level diagram](image)

Figure H1.26: Mass level diagram depending on the various symmetries. Left panel: $SU(3)_L \times SU(3)_R$ symmetry; middle panel: $SU(2)_L \times SU(2)_R$ symmetry; right panel: $SU(2)_V$ symmetry.

H1.8.2 Improving the ultraviolet behavior in baryon chiral perturbation theory

D. Djukanovic, M.R. Schindler, J. Gegelia, S. Scherer

In Ref. [1] we used an old idea by Slavnov [2] who introduced chirally invariant terms with higher derivatives as a regulator of the non-linear sigma model. We included symmetry-preserving higher-derivative terms in the effective Lagrangian of baryon chiral perturbation theory which modify the ultraviolet behavior of the pion and baryon propagators. To regularize the still remaining infinite number of primitively divergent diagrams [2] we apply dimensional regularization. This ensures that all loop diagrams are regulated. The advantage of this approach is that it can be applied to individual Feynman diagrams as well as to integral equations of the few-nucleon sector.


H1.8.3 Improving ultraviolet behavior of BChPT: nucleon-nucleon scattering

D. Djukanovic, J. Gegelia, S. Scherer, M.R. Schindler

The aim of the present work is to develop a systematic approach for calculating physical quantities of the few-nucleon sector within the higher-order derivative formulation of baryon chiral perturbation theory [1]. The main idea is to choose the field variables of the effective Lagrangian so that the ultraviolet divergences occur only in effective potentials of the N-nucleon sector. Only a finite number of diagrams contributes to the potential to any finite order, therefore all divergences can be absorbed in the redefinition of the available parameters of the effective Lagrangian. Our approach is equally well applicable to the two, three, and more nucleon sectors of baryon chiral perturbation theory. We consider in details the nucleon-nucleon scattering in the new framework. Already at leading order we obtain a reasonable description of phase shifts for neutron-proton scattering.