Collaboration X1: Coherent Production of Radiation


Abstract

Experiments have been performed to explore the potential of the low emittance 855 MeV electron beam of the Mainz Microtron MAMI for imaging with coherent X-rays. Transition radiation from a micro-focused electron beam traversing a foil stack served as X-ray source with good transverse coherence. Refraction contrast radiographs of low absorbing materials, in particular polymer strings with diameters between 30 and 450 \( \mu m \), were taken with a polychromatic transition radiation X-ray source with a spectral distribution in the energy range between 8 and about 40 keV. The electron beam spot size had standard deviation \( \sigma_h = (8.6 \pm 0.1) \mu m \) in horizontal and \( \sigma_v = (7.5 \pm 0.1) \mu m \) in vertical direction. As detectors X-ray films were used. The source-to-detector distance amounted to 11.4 m. The objects were placed in a distance of up to 6 m from the X-ray film. Holograms of strings were taken with a beam spot size \( \sigma_v = (0.50 \pm 0.05) \mu m \) in vertical direction, and a monochromatic X-ray beam of 6 keV energy. A good longitudinal coherence has been obtained by the (111) reflection of a flat silicon single crystal in Bragg geometry. It has been demonstrated that a direct exposure CCD chip with a pixel size of \( 13 \times 13 \mu m^2 \) provides a highly efficient on-line detector. The on-line capability allows a minimization of the beam spot size by observing the smallest visible interference fringe spacings or the number of visible fringes. It has been demonstrated that X-ray films are also very useful detectors. The main advantage in comparison with the direct exposure CCD chip is the resolution. For the X-ray film Structurix D3 (Agfa) the standard deviation of the resolution was measured to be \( \sigma_f = (1.2 \pm 0.4) \mu m \), which is about a factor of 6 better as for the direct exposure CCD chip. With the small effective X-ray spot size in vertical direction of \( \sigma_v = (1.2 \pm 0.3) \mu m \) and a geometrical magnification of up to 7.4 high quality holograms of tiny transparent strings were taken in which the holographic information is contained in up to 18 interference fringes.

A novel interferometer has been developed with which magneto optical effects of thin Fe, Co, and Ni foils can be studied at the L\(_{2,3}\) absorption edges. The interferometer consists of two undulators with 10 mm period length. For a magnetized foil, placed between the undulators, \( \Delta \delta(\omega) \) and \( \Delta \beta(\omega) \) of the complex index of refraction \( n_{\pm}(\omega) = 1 - (\delta_0 \pm \Delta \delta(\omega)) + i(\beta_0 \pm \Delta \beta(\omega)) \) will be determined from interference oscillations as function of both, the distance between the undulators and the angle between the polarization planes of the soft X radiation emitted from the first and second undulator. Preliminary results of the Faraday magneto-optical rotation angle \( \psi_F \) in longitudinally magnetized nickel-foils around the L\(_{2,3}\) absorption edges at 872 eV and 855 eV are presented.

To elucidate the quest if the production of parametric X-radiation (PXR) is a kinematical or a dynamical process the radiation from silicon single crystal targets, emitted close to the electron direction, has been studied. The observed interference structures and the narrow band radiation in forward direction shows, that PXR is produced in a dynamical process. In addition, the line width of PXR, measured in backward geometry with a Si single crystal monochromator, was analyzed theoretically. Small angle scattering of the electrons in the crystal leads to a stochastic frequency modulation of the exponentially damped wave train which results in the line broadening. Small angle scattering has been treated with the aid of the Feynman path integral method.
Axial channeling experiments were performed with a low emittance electron beam of 600 MeV energy. The depth dependency of the axial \(< 100\) channeling process was studied for silicon single crystals of thicknesses between 7.9 and 200 \(\mu m\). For the first time it has been demonstrated that the 1.74 keV characteristic Si-K X-radiation is well suited to study the depth dependence of channeling. Because of the small effective projected absorption length of 6.67 \(\mu m\) in our experiment, the method is sensitive to effectively only a thin downstream target layer. The electron-dechanneling function features a sharp decrease within 19 \(\mu m\) and a rather flat tail extending to depths beyond 200 \(\mu m\) with a fraction of about 10-20 \% in the channel. This tail probably originates from rapid dechanneling-rechanneling processes.

Experiments at the 3.41 MeV injector LINAC of MAMI are in progress to study Smith-Purcell radiation from diffraction gratings at frequencies in the THz gap. In these experiments the electron beam is guided close to the diffraction grating over a distance of about 20 cm by means of the magnetic field from a 5 T superconducting solenoid. The accompanying synchrotron radiation due to a helical motion of the electrons has been studied since this component disturbs the interpretation of the Smith-Purcell experiments. A strong coherent enhancement of the emitted power was observed for wavelengths longer than 350 \(\mu m\) which, in principle, enable bunch length measurements.

### X1.1 Introduction

In the research project of the X1 collaboration at MAMI various radiation production processes are of interest. The most important ones are schematically depicted in Fig. X1.1. These are transition radiation (TR), channeling radiation (CR), parametric X-ray radiation (PXR), undulator radiation (UR), and Smith-Purcell radiation (SPR).

![Processes for the generation of coherent radiation with relativistic electrons](image)

At MAMI with UR and TR brilliant photon beams can be produced with energies covering the range between some 100 eV and up to about 50 keV. Brilliance means that a large number of photons from a small source spot size down to the \(\mu m\) range are emitted with high directionality in space. In particular, the hard TR X-ray beam turned out to be comparable in photon flux and brilliance with second generation synchrotron radiation sources [1]. Taking advantage of this fact X-ray phase contrast imaging has been accomplished [2, 3] which is described in section X1.2.
In section X1.3 a novel interferometry principle is introduced with which magneto optical effects of thin Fe, Co, and Ni foils can be studied at the $L_{2,3}$ absorption edges. The interferometer consists of two spatially separated, phase-correlated radiation sources which are undulators with 10 mm period length. Preliminary results of a measurement of the X-ray magnetic circular birefringence, also known as the X-ray Faraday rotation, for nickel at the $L_{2,3}$ edge of 871.9 eV and 854.7 eV will be presented.

If the electron beam strikes a crystal, it emits close to an Bragg angle quasi-monochromatic PXR. This kind of radiation source is amazing for its compactness, since production and monochromatisation of the radiation take place in the same crystal. The expected small spectral line width of PXR would promise an abundance of application possibilities. However, line broadening by multiple scattering of the electrons in the crystal may spoil the superb line width. In section X1.4 our experiments are described addressing the question whether the process of PXR production is of kinematical or dynamical nature. In section X1.5 we present besides a more detailed analysis of previous line width measurements of backward emitted PXR also theoretical investigations using the Feynman path integral method.

In channeling, the charged particle directions are closely aligned with an atomic row or with crystal planes, and their motion is governed by many correlated collisions with crystal atoms. As a result, the particles are steered along strings or planes and CR is emitted. Channeling experiments at MAMI are connected with the feasibility of a crystalline undulator with positrons which was investigated recently in great detail [4]. Since low emittance positron beams are not easily available we commenced experiments with the low emittance 600 MeV electron beam of MAMI, see Ref. [5], which are described in section X1.6.

SPR is generated when a beam of charged particles passes close to the surface of a periodic structure. This type of radiation has been investigated in the optical spectral range with the 855 MeV beam of MAMI [6]. A detailed discussion of the emitted photon number per electron leads to the conclusion that a SPR source is not advantageous in comparison with an UR source for ultrarelativistic beam energies. At present, experiments are being performed at the 3.4 MeV injector LINAC of MAMI to explore the generation of intense SPR in the THz region of the electromagnetic spectrum [7]. The status of the experiment is described in section X1.7.

**X1.2 X-Ray Phase Contrast Imaging at MAMI**

**X1.2.1 Introduction**

The contrast in conventional absorption X-ray imaging is based on the difference in the absorption of the materials constituting the sample. Thin samples of light elements, such as soft tissues and organic materials with $Z \leq 8$, show a weak absorption contrast even at low X-ray energies, i.e., the big deficiency is that the conventional absorption radiography can not distinguish between materials with similar attenuation coefficients. For low Z materials, however, a high contrast could be obtained if the phase shift of the X-rays introduced by the object could be exploited instead of the intensity of the transmitted wave. The enhancement of the contrast is attributed to the fact that, in particular for low-Z materials, the phase shift for X-rays is higher than the absorption of the incident X-rays. Also, for the radiography based on the phase shift mechanism, the absorbed dose is considerably lower in comparison to the conventional absorption radiography, see, e.g., Refs. [8, 9, 10].
X-ray phase contrast imaging can be carried out with various methods, for an overview see the recent Ref. [11]. In particular, it has been pointed out by Wilkins et al. [12] that a very simple experimental setup with a polychromatic X-ray source of good transverse coherence, i.e. a small micro-focused spot, is already sufficient. Information can be supplied by such a method on the sample morphology, i.e. its boundaries, interfaces and location of small features, see e.g. Ref. [13, 14, 15]. If, in addition, the X-ray source emits monochromatic X-rays, holograms can be taken. The experimental setup is similar to that of Gabor in-line holography [16]. In principle, such a setup is rather simple but a highly transverse and longitudinal coherent X-ray source of good intensity and also high spatial resolution detectors are required. Such sources are available at third generation synchrotron radiation sources like ESRF, APS, and SPRING8, and hard X-ray phase contrast imaging, in-line holography and microtomography have been accomplished at these facilities, see, e.g., Refs. [17, 18, 19].

The work presented here exploits the potential of the low emittance 855 MeV electron beam of MAMI to produce X-rays with very good transverse coherence. Our approach is based on transition radiation (TR) production in the X-ray region with a micro-focused electron beam. In subsection X1.2.2 the results of phase contrast imaging with a polychromatic X-ray beam from a TR foil stack with good transverse coherence will be presented. Subsection X1.2.3 deals with our approach toward a hard X-ray in-line holography using monochromatic X-rays. More details can be found in Ref. [2].

**X1.2.2 Refraction contrast radiography**

**X1.2.2.1 Experimental**

The principle of the refraction contrast radiography will be explained by means of Fig. X1.2. The 855 MeV electron beam, with a Lorentz factor $\gamma = 1673$, produces in a transition radiation foil stack a polychromatic X-ray beam which propagates in forward direction in a cone with a typical apex angle of $2/\gamma \approx 1.2$ mrad. The X-ray emission spectrum is shown in the inset of Fig. X1.2. The polychromatic X-rays leave the vacuum system through a polyimide exit window of 120 $\mu$m thickness which is located at a distance of 5.88 m from the foil stack. The beam line is shielded by a concrete wall of 1 m thickness and 3.5 m height to reduce the background in the experimental area. The background originates from electrons which emitted a bremsstrahlung photon in the TR foil stack and left the beam line behind the bending magnet, as well as from the background from the beam dump itself. The objects to be imaged are mounted in air at different distances from the target $x_{so}$ resp. the X-ray film $x_{od}$, with 5.88 m < $x_{so}$ < 13 m and 0 m < $x_{od}$ < 7.12 m. The source-to-detector distance was $x_{sd} = 11.38$ m.

As position sensitive detector the X-ray film Mamoray MR5 II PQ produced by Agfa\(^6\) was used. It is based on silver bromide with an emulsion thickness of $d_f = 12 \mu$m [20]. The exposed X-ray films were processed manually. The X-ray films were digitized with a Nikon film scanner Super CoolScan 4000 ED [21] which has a spatial resolution of 4000 dpi \(^7\) corresponding to a pixel size of $(6.35 \times 6.35)$ $\mu$m\(^2\).

\(^6\)Agfa-Gevaert N.V., B2640 Mortsel Belgium
\(^7\)dots per inch
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Figure X1.2: Schematic diagram showing the experimental setup for refraction contrast radiography. The inset shows the calculated TR spectrum as function of the photon energy for which multiple scattering, electron beam divergence (0.6 mrad) and self absorption were taken into account. The foil stack consists of 30 polyimide foils with a thickness of 25 $\mu$m each, and spacings between the foils of 75 $\mu$m. It was optimized for a photon energy of 33 keV.

X1.2.2.2 Results

An extensive study of the contrast generation as a function of the object-to-detector distance $x_{\text{od}}$ has been performed for strings of different diameters. Fig. X1.3 (a) shows a typical example for a polyamide string with a diameter of about 270 $\mu$m. The calculated absorption contrast for such a string does not exceed about 1%. Therefore, no absorption contrast can be observed with the traditional contact radiography, i.e. for $x_{\text{od}} = 0$, in accord with our measurements. By moving the object away from the detector, the imaging regime is changed from absorption radiography to phase contrast radiography and phase shift is the mechanism to produce the contrast. The contrast appears at the borders of the string where the density gradient reaches its maximum value. An edge contrast can be defined as

$$C_{\text{ref}} = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

with $I_{\text{max}}$ and $I_{\text{min}}$ the maximum and minimum intensity at the edge of the string. As can be seen from Fig. X1.3 (b) the contrast amounts to $C_{\text{ref}} \approx 17.8\%$. The contrast $C_{\text{ref}}$ as function of the object-to-detector distance $x_{\text{od}}$ is shown in Fig. X1.4 as error bars for all measurements.

X1.2.2.3 Discussion

The most interesting feature of the radiograph shown in Fig. X1.3 is that an edge enhancement or phase contrast can be observed with a polychromatic X-ray beam. This fact has been discussed in a number of papers also in connection with the interplay between refraction and diffraction [12, 22]. The general features of refraction contrast imaging will be discussed by means of Fig. X1.4. It can be stated that the distance between object and detector $x_{\text{od}}$ must be at least as large that the wave optical spread of the diffracted X-rays becomes comparable with

8 supplied by Goodfellow
Figure X1.3: Refraction enhanced radiograph of a polyamide string with a diameter of 270 \( \mu \text{m} \) at an object-to-detector distance \( x_{\text{od}} = 5.5 \) m, and a source-to-detector distance \( x_{\text{sd}} = 11.38 \) m. The radiograph was recorded by the X-ray film MAMORAY MR5 II PQ (Agfa). The electron beam current was 6 nA, the exposure time amounted to 60 s. X-ray source sizes were \( \sigma_h = (8.6 \pm 0.1) \ \mu \text{m} \) and \( \sigma_v = (7.5 \pm 0.1) \ \mu \text{m} \) in horizontal and vertical direction, respectively. (a) Radiograph, (b) intensity profile for which 100 vertical pixels were added together to improve the statistics. (c) Normalized intensity profile according to geometrical optics with the following parameters: film and scanner resolution \( \sigma_f = (10.0 \pm 0.4) \ \mu \text{m} \), and the wave optical contribution \( \sigma_{\text{opt}} = \sqrt{\lambda x_{\text{od}} x_{\text{sd}} / (2 \pi \sigma_f)} = 2.3 \ \mu \text{m} \) with \( \lambda = 0.633 \). (d) Same as (c) on the basis of wave optics. For details of the calculations see Ref. [2].

Figure X1.4: Contrast \( C_{\text{ref}} \) for a polyamide string of 270 \( \mu \text{m} \) diameter as a function of the object-to-detector distance \( x_{\text{od}} \). The source-to-detector distance \( x_{\text{so}} = 11.38 \) m was kept constant. Error bars are measurements, crossed circles calculations on the basis of the wave optical model with a beam spot size \( \sigma_s = 7.5 \ \mu \text{m} \) and a total X-ray film resolution and scanner resolution \( \sigma_r = (10.0 \pm 0.4) \ \mu \text{m} \). Stars designate calculations according to geometrical optics.

the detector resolution. Otherwise all interference fringes are blurred and the contrast is low. With increasing object-to-detector distance \( x_{\text{od}} \) the contrast increases about linearly. However, at the same time the projected X-ray spot size on the detector plane increases what worsens
the contrast at larger distances $x_{ad}$. The maximum of the contrast is a function of beam spot size and film resolution. But contrast is not the only figure of merit. It must also be taken into account that with increasing $x_{ad}$ the edge spread increases and the resolution deteriorates. The latter might be undesirable in case that resolution is of importance and nearby features must be resolved.

As an example of the visualization of low-Z objects by the refraction contrast, in Fig. X1.5 the image of a green leaf is shown. In the part labelled with (a) where the leaf is thinner than 1 mm, the visibility of a bundle of vascular tissue (veins) could be resolved with high contrast. In the middle part labelled by (b), the object is about 3 mm thick and contains a bundle of veins. However, the identification of an individual vein is difficult since images from the different veins in the radiograph are overlapping. Such three-dimensional structures may be disentangled by a holographic method some principles of which are sketched in the next section.

Figure X1.5: A refraction contrast radiograph of a part of green leaf *Rumex crispus*. The radiograph was recorded by the X-ray film MAMORAY MR5 II PQ (Agfa). The object-to-detector distance was $x_{ad} = 5.5$ m at a source-to-object distance $x_{so} = 5.88$ m. With these parameters the geometrical magnification was 1.94 times. The electron beam energy was 855 MeV, the electron beam spot size had standard deviations of $\sigma_h = (8.6 \pm 0.1) \, \mu m$ and $\sigma_v = (7.5 \pm 0.1) \, \mu m$ in the horizontal and vertical direction, respectively. The TR foil stack described in Fig. X1.2 was used. The electron beam current was 6 nA, the exposure time 40 s.

X1.2.3 Toward Hard X-ray in-line Holography

In the preceding chapter it has been shown that the transition radiation (TR) X-ray source is well suited for refraction contrast imaging. This chapter deals with the investigation of the possibility of X-ray phase contrast imaging and hard X-ray in-line holography with monochromatic X-rays at MAMI. The good emittance of MAMI allows the preparation of a micro-focus which is a prerequisite of the required transverse coherence of the TR X-ray source. The longitudinal coherence can be achieved by a single crystal monochromator. The basics of in-line holography, the experimental setup, the preparation of the micro-focused electron beam and the results obtained so far will be described in the following.
X1.2.3.1 Basics

A wave emanating from a point source may illuminate an object from which it is scattered. The wave amplitude $E(\vec{r}) = E_0(\vec{r}) + E_{\text{scat}}(\vec{r})$ can be split into the reference wave $E_0(\vec{r})$ and a scattered wave $E_{\text{scat}}(\vec{r}) = a(\vec{r}) \cdot E_0(\vec{r})$. The amplitude ratio can be written as $E(\vec{r})/E_0(\vec{r}) = 1 + a(\vec{r})$. The scattering amplitude $a(\vec{r})$ contains the required information on the object. On a detector screen, such as an X-ray film or a CCD detector, the squared absolute values of the amplitudes $|E(\vec{r})|^2$ and $|E_0(\vec{r})|^2$ are measured from which the contrast image $|E(\vec{r})|^2 - |E_0(\vec{r})|^2$ can be obtained. By division through the reference wave $|E_0(\vec{r})|^2$ the normalized contrast ratio

$$I_{\text{norm}}(\vec{r}) = \frac{|E(\vec{r})|^2 - |E_0(\vec{r})|^2}{|E_0(\vec{r})|^2} = 2 \Re[a(\vec{r})] + |a(\vec{r})|^2 \quad .$$

(X1.2)

can be determined.

The appearance of $2 \Re[a(\vec{r})] = a(\vec{r}) + a^*(\vec{r})$ on the right hand side of Eq. (X1.2) shows that the hologram contains also information on the real part of the scattering amplitude rather than only its absolute value squared $|a(\vec{r})|^2$ which may be referred to as "classical diffraction pattern" of the complementary transmission function of the object [2]. Such classical diffraction patterns are observed in diffraction experiments in which the reference wave is absent, e.g., at diffraction on a slit which is the complementary to an opaque object as, e.g., an opaque wire. While the classical diffraction pattern is rather smooth, see Fig. X1.6 (a), the holographic diffraction pattern oscillates rapidly, see Fig. X1.6 (b). These oscillations have a rather small amplitude and can hardly be seen in a measurement of the hologram such is shown in Fig. X1.6 (c). Much more pronounced oscillations are observed for transparent objects as polymer strings, see Fig. X1.6 (d) and (e) which are maintained in the sum of the classical and the holographic diffraction pattern, see Fig. X1.6 (f). These oscillations contain information on the distance between the object and the detector or the source, and via the refractive index decrement $\delta$ and the absorption $\beta$ also on the bulk of the string. In addition, the hologram contains via the transverse coherence length also information on the beam spot size.

X1.2.3.2 Experimental

The observation of interference patterns as shown, for instance, in Fig. X1.6 requires both, a good transverse and a good longitudinal coherence which can be achieved with a microfocused and monochromatic X-ray beam. These requirements led to an experimental arrangement at MAMI which is schematically depicted in Fig. X1.7. As monochromator a flat single crystal in Bragg geometry is used. The objects to be imaged can be placed between TR radiator and monochromator crystal close to the TR source resulting in a magnification of the object of up to a factor of 7.4 or, alternatively, between monochromator and X-ray detector. The magnification may be of importance to compensate for a moderate detector resolution if, e.g., CCD-chips in a direct exposure mode are used, see below.

For the preparation of a micro-focused electron beam, a low beam emittance in horizontal and vertical directions is of particular importance. The emittance of the MAMI electron beam in horizontal direction is bigger than the emittance in vertical direction because the electrons emit synchrotron radiation in the bending magnets of race-track microtron 3. The horizontal emittance grows rapidly above an electron beam energy of 400 MeV while the vertical emittance still decreases. As a compromise, a beam energy of 600 MeV was choosen for which the
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Figure X1.6: Analysis of the normalized contrast image into distinct patterns for a totally opaque tungsten wire, left column (a), (b) and (c), and for an approximately transparent polymer string, right column (d), (e) and (f). Both wires have the same diameter of 25 µm. The X-ray photon energy is 6 keV (λ = 2.067 Å), the complex refraction index parameters are δ_W = 8.5 \cdot 10^{-5} and β_W = 1.1 \cdot 10^{-5} and δ_P = 7.3 \cdot 10^{-6} and β_P = 2.55 \cdot 10^{-8} for tungsten and polymer, respectively, at this energy. The source-to-object distance is x_{so} = 1.92 m and the object-to-detector distance x_{od} = 11.68 m. Panels (a) and (d) show the classical diffraction pattern |a(z_d)|^2 which is the diffraction pattern of the complementary object, (b) and (e) the holographic diffraction pattern 2Re[a(z_d)] which come about by the interference between the wave front disturbed by the object and the reference wave emanating from the source, (c) and (f) show the normalized contrast images.

The polyimide foil stack to produce transition radiation is optimized for a high X-ray flux at a photon energy of 6 keV at the electron beam energy of 600 MeV. The calculated photon energy spectrum is shown in the inset of Fig. X1.7. The flat silicon single crystal with its surface parallel to the (111) crystal plane acts as a mirror for the TR photons.

Hard X-ray holography requires, like refraction contrast radiography, a two-dimensional resolving detector with a large dynamic range and linear relationship between the incident radiation intensity and the response of the detector. Such conditions can be fulfilled by a charge-coupled device (CCD) or an X-ray film. For the current experiments the CCD system ANDOR DO-434 BN CCD [23] was used. It contains a back-illuminated CCD low noise sensor from Marconi CCD47-10 [24] with 1024 × 1024 pixels of a size of 13 × 13 µm^2. The chip has a good quantum efficiency over a wide spectral range. For X-rays of 6 keV energy it amounts to still about 45%. These features offer the opportunity to use the CCD chip in the direct exposure mode in which the signal is generated by direct energy deposition of X-rays in the sensitive layer of approximately 10 µm thickness.

Direct exposure CCD camera chips have, compared with X-ray films, the big advantage that they have a good linearity over a wide dynamical range, a good signal-to-noise ratio, and that
Figure X1.7: Schematic experimental setup for X-ray in-line holography at MAMI. Shown are the TR foil stack, the single crystal monochromator in a distance of 7.8 m from the target, and a CCD detector or an X-ray film in a distance of 5.8 m from the monochromator. The objects to be imaged can be positioned in distances of 1.88, 4.3, 7.47, 10.78, 12.71 and 13.6 m from the X-ray source. All components are housed in a connected vacuum system to avoid self absorption of the X-rays. The inset shows the calculated TR energy spectrum as function of the photon energy for which multiple scattering, electron beam divergence (0.8 mrad) and self absorption were taken into account. The TR foil stack consists of 25 polyimide foils with a thickness of 12.5 µm which are spaced out by aluminium foils of 100 µm thickness, the latter with centric holes of 2 mm diameter for the passage of the electron beam.

they are on-line capable. The latter fact is very important since contrast or normalized contrast images can easily be generated in which all parasitic background, originating not from the object, can be eliminated. The disadvantage of a moderate spatial resolution in comparison to an X-ray film can be alleviated by a geometrical magnification. In reality, however, the spatial resolution is larger than the pixel size because of so-called split events in which the deposited energy is shared by neighboring pixels.

The X-ray film Structurix D3 from Agfa is a useful detector as well. The main advantage in comparison with the direct exposure CCD chip is its very good resolution. The standard deviation of the resolution was measured to be $\sigma_f = (1.2 \pm 0.4) \mu m$, which is about a factor of 6 better than for the direct exposure CCD chip. The main disadvantage of the X-ray film is the missing on-line capability with the consequence that the generation of normalized contrast images is more complicated.

X1.2.3.3 Measurements and Discussion

X1.2.3.3.1 Investigation of the transverse coherence in horizontal direction To study the coherence in horizontal and vertical direction, radiographs of two polymer strings of the
same thickness of 30 $\mu$m were taken which were mounted horizontally and vertically. The radiographs are shown in Fig. X1.8. Although the beam size in horizontal direction $\sigma_h = (1.7 \pm 0.1) \mu m$ was smaller than $\sigma_v = (3.9 \pm 0.4) \mu m$ in vertical direction, no interference patterns were observed for the vertically mounted polymer string. The maximum contrast, $C_{ref} = (I_{max} - I_{min})/(I_{max} + I_{min})$ for the horizontally mounted string was 64%, while for the vertically mounted one only 11%. The only reasonable explanation for this observation is that the transverse coherence in the horizontal direction is deteriorated by the monochromator crystal. Obviously, in the energy dispersive direction (horizontally) an additional angular divergence is introduced by the crystal. Since the reason of this effect could not be found, all experiments with strings described in this work were performed with horizontally mounted strings.

![Figure X1.8: Two radiographs of a polymer string of 30 $\mu$m diameter. In radiograph (a) the string is mounted vertically, in (b) horizontally. The X-ray source spot size was $\sigma_h = (1.7 \pm 0.1) \mu m$ in horizontal and $\sigma_v = (3.9 \pm 0.4) \mu m$ in vertical direction. The electron beam spot size was checked with the wire scanner before and after the imaging in order exclude a possible shift of the beam spot. The source-to-object distance was $x_{so} = 4.3$ m, and the object-to-detector distance $x_{od} = 9.61$ m. Electron beam current 700 nA, exposure time 1.8 s, 50 frames added up.](image)

Despite of the moderate transverse coherence in horizontal direction holograms could be taken which show also interesting features in horizontal direction. Fig. X1.9 shows a contrast image of a polymer string with a diameter of (150 ± 20) $\mu$m. According to inspection under an optical microscope the string has nearly ideal cylindrical shape. No deformations or impurities inside the string could be observed. However, in the hologram inhomogeneities are clearly visible which may be air bubbles or impurity inclusions with a different density as the string material. The background correction assures that the inhomogeneities do originate from the polymer string and not from dust particles on the monochromator crystal or the detector.

### X1.2.3.3.2 Optimization of the beam spot size

The most important prerequisite for taking high quality holograms is the minimization of the beam spot size. In the first step, the spot size was measured with a tungsten wire of a diameter of $(4.0 \pm 0.4) \mu m$ which was scanned through the electron beam. In particular, the electrical current of the quadrupole doublet in Fig. X1.7 was varied until the scan yielded the smallest spot size. In the next step holograms of polymer strings were taken with the CCD camera and the spot size was estimated from the smallest discernible fringe visibility estimated according to [25]

$$\sigma = 0.31 \frac{x_{so}}{x_{od}} r_{max}$$

(X1.3)
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Figure X1.9: A background corrected hologram (contrast image) of a polymer string with \((150 \pm 20) \mu m\) diameter, supplied by Goodfellow. Source-to-object distance \(x_{so} = 1.88 m\), source-to-detector distance \(x_{sd} = 13.91 m\), corresponding to a magnification of 7.4 times, tilt angle of the object 46°, X-ray source spot size \(\sigma_h = (5.9 \pm 0.1) \mu m\), and \(\sigma_v = (2.6 \pm 0.1) \mu m\), electron beam current 600 nA, exposure time 8.1 s, 50 frames added up.

with \(r_{max}\) the smallest discernable fringe spacing. As already mentioned, a CCD chip allows fast on-line imaging, however, the resolution in the direct exposure mode is limited by the pixel size of 13 \(\mu m\). To achieve a geometrical magnification, the object was placed in a close distance to the X-ray source. In Fig. X1.7 the possible positions of the objects are marked. An example of the spot size measurement by the fringe method is shown in Fig. X1.10 which yielded after optimization a standard deviation \(\sigma_v \approx 1.2 \mu m\).

To exclude a possible influence of the moderate resolution of the CCD detector on the spot size measurement, high quality holograms were taken with the high resolution X-ray film Structurix D3 from Agfa. Fig. X1.11 (a) shows a part of a hologram for a polyamide (Nylon) string. A large number of about 18 interference fringes can be seen, as demonstrated in Fig. X1.11 (b). In this radiograph the main deterioration in the fringe visibility results from the X-ray spot size. The minimum discernable distance between two adjacent fringes is about 25 \(\mu m\) and the estimated X-ray source size is again \(\sigma_v \approx 1.2 \mu m\).

In comparison with the wire scanner measurement which yielded a spot size \(\sigma_v = (0.50 \pm 0.05) \mu m\) the measured values with the fringe method of both, with the direct exposure CCD and the high resolution X-ray film, deviate significantly. This deviation can be explained by the longitudinal depth of the foil stack which amounts to 2.8 mm. For the measured vertical emittance \(\varepsilon_v = 0.52 \mu m mrad\) at the electron beam energy of 600 MeV, and a micro-focused electron beam spot size \(\sigma_v = (0.50 \pm 0.05) \mu m\), the corresponding divergence is 1.04 mrad. For the optimum, when the focus is exactly in the middle of the foil stack, the beam spread within the foil stack amounts to a standard deviation of 1.5 \(\mu m\), in accord with the observation with the direct exposure CCD chip and the X-ray film ⁹.

X1.2.3.3.3 Analysis of holograms for polyamide strings There are two possibilities to analyze holograms of strings. In the first one, calculations on the basis of the Fresnel-Kirchhoff integrals can be performed in which assumptions about the density and morphology of the string are incorporated. The right solution can be found by a systematical trial and error method. An example is shown in Fig. X1.12. Although a good overall agreement between measurement and calculation could be achieved with the assumption of a homogeneous density distribution

⁹The errors of the fringe method may be in the order of 20%
Figure X1.10: Fringe visibility as a function of the X-ray source spot size. Shown are holograms of a polymer string with a diameter of 30 \( \mu \text{m} \) for an X-ray source spot size as measured with the wire scanner of (a) \( \sigma_h = (5.9 \pm 0.1) \mu \text{m} \), \( \sigma_v = (2.6 \pm 0.1) \mu \text{m} \), and (b) \( \sigma_h = (19.1 \pm 0.7) \mu \text{m} \), \( \sigma_v = (0.50 \pm 0.05) \mu \text{m} \). From the smallest discernable fringe spacings \( r_{\text{max}} = 50 \mu \text{m} \) (a) and 25 \( \mu \text{m} \) (b), with Eq. (X1.3) standard deviations \( \sigma_v = 2.4 \mu \text{m} \) (a), and 1.2 \( \mu \text{m} \) (b) result. Source-to-object distance \( x_{\text{so}} = 1.88 \text{ m} \), object-to-detector distance \( x_{\text{od}} = 12.03 \text{ m} \), corresponding to a geometrical magnification of 7.4 times. The angle between string and beam direction amounts to 46°. Electron beam current 500 nA, exposure time 8.1 s per frame, 100 frames added up.

Figure X1.11: (a) Hologram of a polyamide string with diameter of \((150 \pm 20) \mu \text{m}\). Source-to-object distance \( x_{\text{so}} = 1.88 \text{ m} \), object-to-detector distance \( x_{\text{od}} = 11.73 \text{ m} \), corresponding magnification 7.24 times, X-ray source spot size \( \sigma_h = (19.1 \pm 0.7) \mu \text{m} \) and \( \sigma_v = (0.50 \pm 0.05) \mu \text{m} \) as measured with the tungsten wire method. The X-ray film was digitized with an optical microscope with a magnification of 4 in order to maintain a good resolution. Therefore, only part of the hologram was in the field of view. (b) Intensity profile with 200 rows added up.
Figure X1.12: (a) Radiograph of a polyamide string of a diameter of \((150 \pm 20) \mu m\). Source-to-object distance \(x_{so} = 4.3 \text{ m}\), object-to-detector distance \(x_{od} = 9.31 \text{ m}\), corresponding geometrical magnification 3.17 times, X-ray source size \(\sigma_h = (19.1 \pm 0.7) \mu m\) and \(\sigma_v = (0.50 \pm 0.05) \mu m\). The X-ray film was digitized with an optical microscope with a magnification of 4. (b) Intensity profile, 200 rows added up. (c) Calculated intensity profile of the radiograph. The spatial resolution of the X-ray source spot size of \(\sigma_v = (0.50 \pm 0.05) \mu m\), of the film of \(\sigma_f = (1.2 \pm 0.4) \mu m\), and the optical resolution of \((\sigma_{sc,4} = 4.1 \pm 0.1 \mu m)\) were incorporated into the calculations.

within the string, in detail significant differences in the interference pattern close to the boundaries can be recognized. These differences may indicate a density gradient at the periphery of the string. Refinements of the string model are required to further investigate the origin of these differences.

The second possibility is based on reconstruction algorithms to find the phase profile produced by the transparent object. One of these is the modified Gerchberg-Saxton algorithm [26]. It is an iterative method with which the phase information can be found from two holograms taken at different distances between object and detector. Therefore, holograms of a polyamide string with a diameter of 30 \(\mu m\) were taken at different object-to-detector distances. However, this issue went beyond the scope of this explorative experimental work and is subject of ongoing investigations.

X1.2.4 Conclusions

Phase contrast radiography has been accomplished with an external 855 MeV electron beam using broad band transition radiation X-rays with a mean photon energy \(<h\omega> \approx 20 \text{ keV}\) and a micro-focus with standard deviations of typically \(\sigma_h = 8.6 \mu m\) and \(\sigma_v = 7.5 \mu m\) in horizontal and vertical direction, respectively. In-line holograms of polymer strings were taken with a low
emittance 600 MeV electron beam using narrow band transition radiation X-rays with a photon energy of $\hbar\omega = 6$ keV and a micro-focus with a standard deviation of typically $\sigma_v = 1.2 \mu m$. High quality holograms were obtained with high resolution X-ray films and a direct exposure cooled CCD camera ship. The advantage of the former is the very good spatial resolution, that of the latter its on-line capability.

An X-ray beam spot with micro-dimensions can be prepared directly with the micro-focused external electron beam via transition radiation production in a foil stack. Objects to be investigated can be placed in close distance to the small X-ray beam spot. This has the advantage that a large geometrical magnification of up to a factor of 10 can easily be achieved in our relatively small experimental area. The disadvantage of the transition radiation X-ray source is its contamination with high energy bremsstrahlung photons.

Typical electron beam charges required to capture a single image are about 0.3 $\mu C$ for phase contrast radiographs with broad band polychromatic X-rays and an X-ray film as detector, and some nC for a cooled CCD chip. In-line holograms with narrow band X-rays require about 500 $\mu C$ using a high resolution X-ray film, and 5-10 $\mu C$ for the CCD detector.

### X1.3 A Soft X-ray Interferometer for the Investigation of Magneto-Optical Effects

#### X1.3.1 Introduction

The change of the polarization state of light at the passage through magnetized materials are called magneto-optical effects. These effects were observed in rare earth elements and transition metals as Fe, Co, Ni close to the “white line” which are strong absorption lines resulting from an allowed dipole transition between the $2p_{3/2}$ ($2p_{1/2}$) core state and empty 3d valence states. These transitions are accompanied by a strong X-ray magnetic circular dichroism (XMCD) effect which can be used to probe the magnetic properties of the material [27]. With XMCD spectroscopy, pioneered by Schütz and coworkers [28], the difference in the absorption of left and right circular polarized light was measured. Connected with the XMCD is the X-ray magnetic circular birefringence (XMCB). Both effects can be described with the complex index of refraction $n_{\pm} = 1 - (\delta_0 \pm \Delta \delta) + i(\beta_0 \pm \Delta \beta)$ for the two helicity states of the radiation.

At a measurement of XMCD, thickness variations of the foil in a transmission measurement hamper the accuracy in the determination of $\Delta \beta(\omega)$ with the XMCD spectroscopy, in particular near strong absorption lines. It has been shown in Ref. [30], that such thickness effects are less important for a measurement of XMCB, i.e. the refractive index decrement $\delta_{\pm} = (\delta_0 \pm \Delta \delta)$. In this work, a novel method for the measurement of the real part decrement $\delta_{\pm}$ and the absorption index $\beta_{\pm} = (\beta_0 \pm \Delta \beta)$ of thin self-supporting foils in the energy range between 50-1500 eV is being developed. This method is a further development of a previous experimental setup with which the refractive index decrement $\delta_0$, and absorption index $\beta_0$ was measured for nickel at L$_{2,3}$ edges with unmagnetized foils [29]. In the following the novel soft X-ray interferometer is described and preliminary results are presented.
X1.3.2 Experimental

The basic features of the interferometer, shown schematically in Fig. X1.13, were already described in the last annual report [31]. The soft X-ray radiation is produced by a collinear arrangement of two identical undulators with 10 periods each. The undulator period amounts to 12 mm. The second undulator can be both, moved along and also be rotated around the electron beam axis. These possibilities allow the production of radiation with an arbitrary polarization state between linear and circular. In addition, the undulator gap can be varied allowing an online variation of the photon energy via the undulator parameter $K$. The radiation is monochromized by a grazing incidence spectrometer employing a grating of variable line density [29, 32] and detected by a windowless CCD-detector [23] with 1024 x 1024 pixels, pixel size $13 \mu m \times 13 \mu m$ and sensitive layer of about $10 \mu m$ thickness.

![Figure X1.13: Experimental setup. Shown are the undulators $U_0$ and $U_r$ and the magnetized foil between them. The second undulator can be both, moved along and rotated around the electron beam axis. These possibilities allow the analysis of the elliptically polarized X radiation behind the magnetized foil.](image1)

![Figure X1.14: Experimental setup of the novel undulator interferometer. The undulator gap can be varied online. This way the undulator parameter $K$ and, in turn, the photon energy be changed. Note that the electron beam traverses a vacuum tube of only 3 mm in diameter over a length of about 1 m.](image2)

This interferometer allows also a measurement of $\Delta\delta(\omega)$ without any polarization filter. The magnetized sample foil with the magnetization direction parallel or antiparallel to the electron beam axis is positioned between the two undulators. Due to the XMCD and XMCD effect the linear polarized light from the first undulator suffers a helicity dependent rotation and
absorption resulting in elliptical polarized light. The second undulator which can be rotated
around the electron beam axis acts as an analyzer. Maximum visibility of intensity oscillations
as function of the distance between the undulators is observed if the angle of rotation of the
second undulator $\Psi$ corresponds to the Faraday magneto-optical rotation angle $\psi_F$, i.e. the
inclination angle $\psi_F$ of the ellipse the electrical field vector traces out with respect to the
polarization plane of the linear polarized light in front of the foil. A detailed analysis shows
that $\psi_F$ is related to $\Delta \delta$ by the relation $\psi_F(\omega) = t_0 (\omega/c) \Delta \delta(\omega)$.

Model calculations have shown that beam spot sizes with standard deviations of 220 $\mu$m hor-
vizontally and 70 $\mu$m vertically are required to achieve a good visibility of the intensity oscil-
lations if the distance between the undulators is varied. The beam spot sizes were measured
with wire scanners positioned in the two chambers in front and behind the undulators. Stan-
dard deviations of $\sigma_x = 220 \mu$m and $\sigma_y = 77 \mu$m were measured which correspond to standard
deviations of the beam divergence of 22 $\mu$rad and 5 $\mu$rad which were calculated with measured
emittances $\epsilon_x = 4.9 \, \pi \, m \, \text{rad (horizontal)}$ and $\epsilon_y = 0.4 \, \pi \, m \, \text{rad (vertical)}$ for an electron
beam energy of 690 MeV. With a focus at the target foil, the electron beam can be regarded in
the entire undulator setup in good approximation as parallel. The electron beam must be ad-
justed vertically to the magnetic middle of the undulators since already a deviation of 300 $\mu$m
diminishes the calculated visibility from 76 % to 56 %. This fact requires a sophisticated beam-
monitoring system which was also already described in the last annual report [31]. In the mean
time, a computer program was written to stabilize the electron beam vertically and horizon-
tally. The currents of dipole magnets in front of the undulators were controlled according to
the deviation of the beam position which was measured with rf-resonators in front and behind
the undulator-setup. For this purpose a series of beam diagnostic pulses with a width of 10 ns
and a frequency of 10 kHz were applied every 14 s once.

Figure X1.15: Measurement of the resolution of the spectrometer. (a) CCD image with a 200 mm long
vessel filled with 2 mbar neon in front of the detector. Clearly recognizable are the 1s-3p and 1s-4p
absorption lines of neon. The displayed area corresponds to $\Delta h\omega \times \Delta \theta_x = 54 \, eV \times 0.73 \, \text{mrad}$. (b)
Intensity profile after projection of the angular interval $-0.355 \, \mu \text{rad} \leq \theta_x \leq 0.355 \, \mu \text{rad}$ on the $h\omega$ axis.
Full lines are best fits. The energy difference between the 1s-3p and 1s-4p transitions of 1.63 eV [33, 29]
has been used for the purpose of energy calibration.

Additional measurements were performed to determine the energy-resolution of the grating-
spectrometer. For this measurements a chamber of 200 mm length was placed in front of the
CCD-detector which was filled with neon at a pressure of about 2 mbar. The measurement is
COLLABORATION X1: COHERENT PRODUCTION OF RADIATION

Figure X1.16: Images of the CCD chip with the nickel foil. (a) Image taken with only undulator U₀. The nickel L₃ and L₂ absorption lines at 855 eV and 872 eV, respectively, are marked. (b) Image with both undulators. Clearly recognizable are intensity oscillations in the energy dispersive vertical direction. The displayed area corresponds to \( \Delta \omega \approx 54 \text{ eV} \times 0.73 \text{ mrad} \). Exposure time 6 s at an electron beam current of 566 nA.

shown in Fig. X1.15. A line-width of 0.37 eV (FWHM) was measured for the 1s-3p transition at \( h\omega = 867.05 \text{ eV} \) [33] which corresponds after de-convolution from the natural line-width \( \Gamma = 0.27 \text{ eV} \) [34, 35] to an energy resolution of \( \Delta h\omega = 0.27 \text{ eV} \) (FWHM) or \( h\omega/\Delta h\omega = 3211 \) of the spectrometer.

X1.3.3 Preliminary results

With the above described experimental setup several test measurements at an electron beam energy of 690 MeV were performed. The sample used in these investigations was a nickel foil of 72.5 nm thickness, placed in a homogeneous longitudinal magnetic field of 1.0 T. For a nickel foil with the magnetic field in direction of the surface normal, the saturation magnetization is reached at an external field of 0.9 T, as concluded from measurements [36]. Fig. X1.16 shows images of the directly exposed CCD chip.

As already mentioned, a translation of the second undulator along the electron beam axis leads to intensity oscillations. Various measurements were performed for different rotation angles \( \Psi \) of undulator \( U_r \) without and with the nickel foil. Measurements without the nickel foil are shown in Fig. X1.17 (a). As expected, the intensity oscillations have the largest amplitude at a rotation angle \( \Psi = 0^\circ \) of undulator \( U_r \) and vanish at \( \Psi = 90^\circ \). The visibility \( V = (I_{\text{max}} - I_{\text{min}})/(I_{\text{max}} + I_{\text{min}}) \) has been evaluated for all 1024 x 1024 pixels of the CCD chip. Fig. X1.18 shows the resulting so called ”visibility-map”.

For measurements with a magnetized nickel foil, Fig. X1.17(b), the maximum visibility is observed at a finite rotation angle \( \Psi \) of undulator \( U_r \) which corresponds to the Faraday rotation angle \( \psi_F \). In Fig. X1.17(c) the visibility is shown for rotation angles of \( U_r \) in the interval \(-107^\circ < \Psi < +107^\circ \). This figure reveals that the visibility with the magnetized nickel foil is attenuated and shifted by several degrees with respect to the reference curve without a nickel foil. Preliminary results of the Faraday rotation angle \( \psi_F \) are shown in Fig. X1.19 as function of the photon energy around the the L₃ and L₂ absorption edges of nickel. From these measurements the refractive index decrement \( \Delta \delta \) will be extracted in a forthcoming analysis.
Figure X1.17: Intensity oscillations and visibility. Shown are the intensities as function of the relative distance $\Delta d$ between the undulators for rotation angles $\Psi = 0^\circ, 45^\circ$ and $90^\circ$ of undulator $U_r$ (a) without and (b) with a magnetized sample foil. Small regions $\Delta\omega \times \Delta\theta = 0.05$ eV $\times 0.71$ $\mu$rad were selected at the CCD chip. Panel (c) shows the visibility $V = (I_{\text{max}} - I_{\text{min}})/(I_{\text{max}} + I_{\text{min}})$ as function of $\Psi$ without and with the nickel foil at photon energies of $855$ eV (without foil), $854.57$ eV (middle curve) and $855.21$ eV (lower curve). Full lines represent best fits with a $j \cos(\Psi + \psi_F)$ function, with $\psi_F$ the Faraday rotation angle.

Additional information is contained in the the phase shift of the intensity oscillation as shown in Fig. X1.17 (b). Calculations show that phase shift and amplitude close to the minimum of the visibility should be sensitive to the absorption index part $\Delta\beta$. The future analysis will exhibit with which accuracy the complete helicity dependent complex index of refraction $n(\omega)$ can be determined from the measurements presented here, including also a "phase shift ma", without knowing the degree of the polarization of the light.

In conclusion, a novel interferometer has been developed with which it should be possible to measure $\Delta\delta(\omega)$ and $\Delta\beta(\omega)$ of the complex index of refraction $n(\omega) = 1 - (\delta_0 \pm \Delta\delta(\omega)) + i(\beta_0 \pm \Delta\beta(\omega))$. At the present stage of the experiment the Faraday magneto-optical rotation angle $\psi_F$ in magnetized nickel-foils was measured at the L$_{2,3}$ absorption edges at $872$ eV and $855$ eV, respectively.

### X1.4 Measurement of Forward Diffracted Parametric X-Ray Radiation

Parametric X-ray radiation (PXR) or quasi–Cherenkov radiation is produced when a relativistic electron traverses a single crystal, and the wave vector $\vec{k}_v$ of the virtual photon associated with the electron field, the reciprocal lattice vector $\vec{H}$ of a specific crystal plane, and the wave...
Collaboration X1: Coherent Production of Radiation

Figure X1.18: Visibility map without a nickel foil. The visibility is shown for a horizontal observation angle interval $\Delta \theta_x = 0.5 \, \text{mrad}$ and the whole energy range covered by the CCD chip. The visibility decreases rapidly at off-axis observation angles $|\theta_x| > 20 \, \mu\text{rad}$. Therefore, at the data analysis procedure only a small angular range of $\Delta \theta_x = 7.1 \, \mu\text{rad}$ was taken which corresponds to a width of about 10 pixels of the CCD chip.

Figure X1.19: Faraday rotation angle $\psi_F$ as function of the photon energy for a 72.5 nm thick self supporting magnetized Ni foil around the L$_3$ and L$_2$ absorption edges. Graphics (a) and (b) show results with the magnetization direction parallel and antiparallel to the electron beam direction, respectively.

vector $\vec{k}_r$ of the emitted X-ray nearly fulfil the diffraction condition $\vec{k}_r + \vec{H} \cong \vec{k}_r$. For ultra-relativistic electrons the wave vector $\vec{k}_r$ nearly coincides with the electron velocity vector $\vec{v}$ and the emission process can also be understood as diffraction of virtual photons by the crystal. The production mechanism of PXR in such a medium with three-dimensional periodic permittivity was extensively studied both theoretically and experimentally by a large number of researchers. For an overview of the theoretical and experimental work up to the year 1997 see e.g. Ref. [37] and also Ref. [38, 39, 40, 41] for the recent works.

Up to now it could not be decided experimentally whether PXR emission is a kinematical or dynamical process. In the kinematical picture only the PXR wave is required while in the dynamical picture an additional forward diffracted wave (FD-PXR) must occur which is associated to PXR and which is emitted close to the direction of the electron propagation. The reason has been discussed by Nitta [42]. He showed that the first-order approximation of the dynamical calculation gives the kinematical expression. Extremely accurate absolute intensity measurements would be required to figure out a difference. Baryshevsky [43] proposed to search for the
predicted FDPXR wave. Similar proposals have also been communicated by Nasonov [44, 45]. In Ref. [46] the observation of narrow FDPXR structures from a 410 µm thick tungsten single crystal at photon energies of 28.3 and 40 keV is reported.

At MAMI experiments were performed for the search of the forward diffracted wave (FDPXR) in single silicon crystals of various thicknesses [47]. The basic idea of the experiment will be explained by means of Fig. X1.20. A silicon single crystal target was positioned in such a way that the PXR reflex at the photon energy $\hbar\omega_0 = 10.554 \text{ keV}$ is located at twice the Bragg angle of $\Theta_0 = 10.797^\circ$ in the horizontal plane of drawing. The radiation in forward direction close to the electron direction was analyzed with a flat silicon single crystal monochromator in combination with a pn-CCD camera as a position sensitive and energy resolving photon detector. The quasi-monochromatic FDPXR peak energy matches with the energy of the analyzer crystal at only one specific observation angle $\theta_x$. Since the reflecting power ratio of the crystal monochromator exhibits energetically a narrow band characteristics, quasi-monochromatic intensity structures emitted from the target crystal can be detected by this experimental arrangement.

Experiments were performed with target crystals of varying thickness and for different photon energies. As an example the intensity distributions of the experiment with 58 µm and 1 mm crystal thickness are shown in Fig. X1.21. The most striking feature are the structures which move across the pn-CCD detector if the rotation angle $\psi_x$ of the target crystal around the vertical $y$ axis is varied. Moreover, inspection of Figure X1.21 reveals that the structures observed for the thin and thick targets are correlated with each other and, therefore, must have a common physical origin.

Our analysis [47] shows that the common origin can be found in a resonance in the dispersion surface of the electron field which gives rise to the FDPXR amplitude. Its pole and close by zero influence via causality also the optical properties of the crystalline matter far off the resonance, in particular also the real part of the refractive index which creates the transition radiation interference pattern shown in Fig. X1.21. Calculations on the basis of this model, which are depicted in Fig. X1.22, are in accord with the experimental observations. The structures observed for thin targets originate according to this model nearly entirely from the interference of the transition radiation amplitudes generated in the entrance and exit interfaces of the single crystal target. In contrast, the narrow peaks observed for the thick target, see Fig. X1.21, right panel, originate from the additional FDPXR amplitude. Therefore, it must be concluded
Figure X1.21: Measurements at photon energy of 10.554 keV and target thickness of 58 µm (left) and 1000 µm (right). Shown are intensity distributions summed over all 64 rows of the pn-CCD detector as a function of the column number. From the upper to the lower panel the rotation angle $\psi_x$ of the target crystal was varied in steps $\delta \psi_x = 0.5894$ mrad. Beam current 53.5 nA, exposure time 600 s. Left panel: Beam spot size about 500 µm (FWHM) horizontally and 434 µm (FWHM) vertically. The destructive interference fringes can clearly be recognized. Right panel, upper curves: Beam spot size about 500 µm (FWHM) horizontally and 434 µm (FWHM) vertically. Right panel, lower curves: Reduced beam spot size 114 µm (FWHM) horizontally and 200 µm (FWHM) vertically.

that within the framework of this model both phenomena have the same physical origin. Both observations, with thin and thick targets, prove that generation of PXR in a single crystal is a dynamical process.

X1.5 On the Line Width of Parametric X-Ray Radiation

X1.5.1 Measurements of the line width at backward emission

Theoretical descriptions of PXR predict a very narrow line width of less than a few meV. Actually, if it is assumed that a charged particle passes a semi-infinite thick and perfect single crystal on a straight trajectory without scattering and if self absorption of the photons in the crystal can be neglected, simply a $\delta$-function results for the line shape. If self absorption is taken into account, the line width is determined from the Fourier transform of the emitted exponentially damped wave train which is a Lorentzian with a width [48]

$$W_{nat} = \frac{|\chi_0'|}{2 \sin^2 \theta_0} \cdot h\omega_0. \quad (X1.4)$$

The quantity $\chi_0'$ is the imaginary part of the mean dielectric susceptibility $\chi_0 = \chi_0' + i\chi_0''$. As an example, according to Eq. (X1.4) this so-called "natural line width" $W_{nat}$ is for the (444) reflection of silicon at backward emission, i.e. for $\theta_0 \simeq \pi/2$, with $h\omega_0 = 7908$ eV and $\chi_0'' = -3.74 \times 10^{-7}$ [49] $W_{nat} = 1.48$ meV. It is interesting to note that the corresponding Darwin-Prins curve has a width of $W_{DP} = 38.5$ meV and is a factor of 26 broader, see also Fig. X1.23. In
view of the fact that such a narrow bandwidth source could be of extreme interest for many applications, a number of experiments were performed to determine the line width of PXR. Measurements at the low electron beam energy of 6.8 MeV results in a line width of 48 eV for a 55 µm thick diamond crystal at a photon energy of 8.98 keV [50]. This rather large line width originates from the multiple scattering of electrons in the crystal. With the critical absorber technique experiments have been performed at MAMI at an electron beam energy of 855 MeV [51]. Upper limits of the line width of 1.2 eV and 3.5 eV have been determined for the (111) and (022) reflections of silicon single crystals at photon energies of 4966 eV and 8332 eV. These limits originate mainly from geometrical line broadening effects.

In backward geometry geometrical line broadening contributions, originating from the angular spread of the electron beam and small angle scattering of the electrons in the crystal, minimize. To investigate whether under these conditions line widths smaller as the Darwin-Prins values can be reached, experiments have been performed at MAMI which are described in more detail in Ref. [52] The results are summarized in Fig. X1.23.

It has been discussed in Ref. [39] that the small angle scattering of the electron in the Coulomb potential of the crystal atoms results in a stochastic change of the electron direction which leads to a stochastic frequency modulation of the exponentially damped wave train of PXR. As a consequence, the PXR line broadens. The scattering distribution function was approximated by a Gaussian. This approximation may not be appropriate if the electron enters the crystal

---

Figure X1.22: Results of simulation at photon energy of 10.554 keV and target thickness of 58 µm (left) and 1000 µm (right). Shown are number of photons ΔN per pixel and electron for one row of the pn-CCD detector as a function of the column number. >From the upper to the lower panel the rotation angle of the target crystal ψx was varied in steps of Δψx = 0.5894 mrad. The beam spot sizes of the experiment and scattering of the electron beam were taken into account. The residual interference oscillations originate from the interference of the remaining 4 % amplitude created at the entrance interface with the amplitude at the exit interface of the crystal. These oscillations are smoothed out in a real experiment in which over several or all rows of the pn-CCD detector is summed.
close to a channeling axis or a channeling plane. In such cases Monte-Carlo simulations of the scattering process were applied to obtain the PXR wave train. However, it is interesting to notice that calculations according to Ref. [39] as well as also Monte-Carlo simulated line shapes [53], which are both shown also in Fig. X1.23, are in good agreement with the measurements.

![Graph showing measured and calculated line widths for various (nnn) reflections for a silicon single crystal of 525 µm thickness. Open circles: measured PXR line width, full squares: natural PXR line widths for straight electron trajectories, open squares: Darwin Prins width, triangles: line widths calculated with the analytical model of Ref. [39], and stars: Monte-Carlo simulations [53].]

**X1.5.2 Calculation of line broadening by the Feynman path integral method**

In this subsection the effects of small angle multiple scattering of relativistic electrons on the line width of PXR will be treated with the aid of the Feynman path integral method. It will be assumed that the electron impinges to one of the crystallographic axes of a single crystal, the $z$ axis in Fig. X1.24, at an angle $\psi$ which is larger than the critical channeling angle $\psi_c$.

Small angle scattering of relativistic electrons in the crystal leads to a small change of the intersection angle of the electron with the different crystallographic planes which, in turn, leads to a destruction of the constructive interference of the reflected waves from the various planes. To obtain the shape of the PXR line, the spectral-angular density of PXR has been calculated for a relativistic electron which moves along a trajectory $\mathbf{r}(t)$ through a medium with nonuniform dielectric permittivity $\varepsilon_\omega(\mathbf{r}) = 1 + \varepsilon'_\omega(\mathbf{r})$. Near a PXR lines, i.e. for $\omega \approx \omega_n$, the result is

$$
\frac{d^2W}{d\Omega d\omega} = \frac{e^2 \omega_n^2}{4\pi^2 a^2} |e'_{\omega,g}|^2 \eta^2(\theta, \psi) L^2 F(\omega - \omega_n, L),
$$

with

$$
F(\omega - \omega_n, L) = \frac{1}{L^2} 2 \Re \int_0^L dt \int_0^t dt' e^{i(\omega - \omega_n)\Phi(t, t')},
$$

and

$$
\Phi(t, t') = \exp \left\{ \frac{i\omega_n}{2} \int_t^{t'} d\tau v_\perp^2(\tau) - i\omega_n \beta \int_t^{t'} d\tau v_\parallel(\tau) \right\}.
$$
Figure X1.24: Definition of relevant quantities. The symbol $\mathbf{v}$ represents the velocity vector of the incoming electrons, $\mathbf{g}$ is the reciprocal lattice vector, $\mathbf{k}$ the wave vector of the radiated photons, $\psi$ the incidence angle of electrons with respect to the $z$ axis, $\theta$ the angle between $\mathbf{k}$ and $\mathbf{g}$, $N$ the number of lattice planes, and $a$ the separation of the lattice planes.

The function $F(\omega - \omega_n, L)$ describes via the function $\Phi(t, t')$ the line broadening of PXR due to multiple scattering. The quantity $\eta(\theta, \psi)$ is a function of the angles $\theta$ and $\psi$, $\chi = 1 + \cos(\theta + \psi)$, $\beta = \sin(\theta + \psi) - (1 + \cos(\theta + \psi)) \sin \psi$, $e$ the particle charge, $a$ the distance between crystallographic planes, and $L$ the thickness of the crystal. The energy of a radiated photon is given by $\omega_n = v g \cos \psi (1 + v \cos(\theta + \psi))^{-1}$.

Equation X1.5 has been obtained on the basis of the Maxwell equations for the interaction of the electron field with the 3-dimensional periodical dielectric permittivity of the crystal taking into account the finite number of crystallographic planes. It has been assumed that the PXR originates from the reflection of the electron field from crystal planes. The small angle scattering of relativistic electrons in the crystal enters via the function $\Phi(t, t')$. If scattering of the electrons is neglected this function reduces to 1. In this case, the line shape in semi-infinite crystal is described by a Dirac $\delta$-function since self absorption of the X-rays in the crystal have been neglected in this theory.

In the case when the electron enters the crystal at a small angle $\psi (\psi > \psi_c)$ with respect to one of the crystallographic axes and the radiation is emitted in backward direction (see [58]), $\chi$ and $\beta$ can be expanded in powers of small angles $\theta$ and $\psi$. With $\chi = 2$ and $\beta = \theta - \psi$ the result of Ref. [60, 61] is obtained.

The function $v_\perp(\tau)$ defines the random small angle scattering of the electron in the crystal. At an incidence angle $\psi (\psi > \psi_c)$ the scattering process of relativistic electrons in the crystal can be approximated by a Gaussian [59]. In this case the relativistic electron scattering occurs mainly along the azimuthal angle $\phi$ in the plane, normal to the $z$ axis [62] and a redistribution of electrons over this angle occurs as a result of the coherent scattering by atomic strings of the crystal, which are oriented parallel to the $z$ axis. The mean square of multiple scattering angle is given by $\bar{\theta}^2 = q_c L$ [62], where $q_c$ is the mean square of the coherent scattering angle per
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unit length. Thermal displacements of the atom position in lattice result in a non-coherent scattering, and the mean square scattering angle approaches the value $\overline{\theta^2} = q_a L$ of an amorphous medium. Thus, particle trajectory deviations along the $y$ axis, which is oriented perpendicularly to the $(z,v)$ plane, causes both, coherent and non-coherent scattering in the crystal, and the mean square scattering angle is given by $\overline{\theta^2} = q_a L$ with $q_c = q_c + q_a/2$. A particle deviation along the $x$ axis is mainly caused by non-coherent scattering with a mean square scattering angle of $\overline{\theta^2} = q_x L$, where $q_x = q_a/2$. In Bragg geometry, both, coherent and non-coherent scattering processes can be described by Gaussians \[62\] with different mean square values of the scattering angle for deviations along the $x$ and $y$ axes. Since the scattering process is described by a Gaussian also the $\Phi(t,t')$ function is a Gaussian and the Feynman path integral method \[63, 64, 65\] can be used to average the spectral-angular density of PXR with respect to the random scattering angle $\nu_\perp(\tau)$. The final result of this averaging procedure describes the line shape which is given by

$$F(\omega - \omega_n, L) = 2 \int_0^1 dz \int_0^z du \cos(\chi(\omega - \omega_n) L) e^{-\mu^2 \sigma^2 u z}.$$  \hspace{1cm} (X1.8)

This formula has been obtained in first order approximation in which the main contribution to $F(\omega - \omega_n, L)$ originates mainly from non-coherent scattering processes. The $q_x$ value deviates by less than $10 \div 15\%$ from the corresponding value for an amorphous medium $q_a^x$ \[62\]. The $q_a^x$ value in the multiple scattering theory of charged particles in an amorphous medium is defined by the relation \[66\] $q_a^x = \eta (14 \text{MeV}/\epsilon)^2/L_R$, where $\epsilon$ is the particle energy in MeV, $L_R$ is radiation length, for a silicon crystal $L_R \approx 10\text{cm}$, and $\eta \approx 0.5$.

![Figure X1.25: Numerical calculation of the PXR line width. For details see text.](image)

The numerical evaluation of this equation X1.25 is presented in Fig. X1.25 for similar conditions as for the experiment of Ref. [57]. As shown in this figure, the upper limit of the PXR line width $\Delta \omega$ for the $<111>$ reflection of a silicon single crystal (thickness $L = 128\mu\text{m}$) at a photon energy of $\omega \approx 5\text{keV}$ and an electron energy of $\epsilon = 855\text{MeV}$ is about $\Delta \omega \approx 400\text{meV}$. This value is much larger than the natural line width $\Delta \omega \approx 1.6\text{meV}$ for an absorption length $L = 128\mu\text{m}$. This value amounts to approximately 30% of the measured line width $\Delta \omega = 1.2\text{eV}$ for the MAMI experiment [57].
In the extreme Bragg case of backward PXR emission, for which \( \chi = 2 \) and \( \mu = \theta - \psi / \sqrt{2q_x L} \) holds, the PXR line width at the experimental conditions described in Ref. [58], i.e. for a silicon single crystal with a thickness of \( L = 525 \mu m \) and the \( < 333 > \) reflection, is \( \Delta \omega = 20 \text{meV} \) which is much larger than the natural line width \( \Delta \omega_0 : 4 \text{meV} \) for \( L = 525 \mu m \).

The numerical calculations of the line shape \( F(\omega - \omega_n) \) for both cases of PXR emission shows that the multiple scattering process of electrons in the crystal makes an essential contribution to line width of PXR. It should be noted that the theoretical numbers are smaller than the experimental values [57, 58]. The reason is that self absorption of the X-rays in the rather thick crystal makes an essential contribution. The calculations presented here concentrated mainly on the problem of the influence of multiple scattering of relativistic electrons in the crystal on the line width of PXR using the Feynman path integral method. For a more complete line shape theory the photon absorption effects must be included in the spectral-angular radiation density of PXR.

**X1.6 Axial Channeling Experiments at Silicon Single Crystals**

**X1.6.1 Introduction**

In channeling, the charged particle directions are closely aligned with an atomic row (string) or with crystal planes, and their motion is governed by many correlated collisions with the crystal atoms. As a result, the particles are steered along strings or planes and all cross-sections associated with the interaction of charged particles with matter are substantially changed in comparison to amorphous matter. Recently, the feasibility of a crystalline undulator has been theoretically investigated in great detail, e.g. [67, 68] and references cited therein. Such a device combines both, channeling of positrons through periodically bent crystals with emission of channeling radiation, and emission of undulator radiation to achieve lasing in the very hard X-ray region of MeV range. The motivation of the experiments described in this section is connected with our interest to realize such a gamma-laser experimentally. We felt that a thorough understanding of the basic underlying process, the planar channeling of positrons in the (110) transverse potential of a silicon single crystal is mandatory. Although investigated theoretically in detail, the dechanneling in bent crystals should also be explored experimentally. Since low emittance positron beams are not easily available we commenced experiments with the low emittance 600 MeV electron beam of MAMI. Only very little is known experimentally on the question at which depth in the crystal an axially captured electron dechannels, i.e. leaves the axial channel. The latter is connected to the question over which distances an electron can be steered in a string of the crystal which might also be bent.

In principle, channeling of electrons can be studied by any kind of close encounter processes. To these belong scattering of electrons off the atomic nuclei, including coherent scattering on many atoms in a string, and associated energy losses by emission of bremsstrahlung photons. Detected can be the scattered electrons, directly or by electromagnetic shower production, the resulting recoil ions, directly or by their radioactivity in case of nuclear reactions [69], Møller electrons knocked out from inner shells, or the subsequently emitted characteristic X-rays [70, 71]. In Fig. X1.26 the impact parameter dependence of the K-shell ionization process in silicon atoms is depicted. The figure demonstrates that this process is sensitive to small impact parameters less than about 0.15 Å.
Impact Parameter $b$ [Å]

![Figure X1.26: K-shell ionization probability as function of the impact parameter $b$ for silicon atoms and 600 MeV electrons. The electron $e^-$ creates a K hole by ejection of a recoil (Møller) electron. The K-shell ionization probability, as calculated according to [72, Eqs. (5.10) and (5.21)], shows that characteristic K X-ray emission is sensitive to small impact parameters $b < 0.15$ Å.](image)

With detection of scattered high-energy electrons only averaged information over the target thickness on the channeling process can be obtained. For the investigation of the dechanneling process, the fraction of electrons remaining in a channel must be measured as function of the traversed crystal thickness. To reach this objective, it is much more favorably to choose radiation components which feature a strong absorption characteristics in the target, and to position the detector downstream the target. It is obvious that for low energy characteristic X-rays from silicon with an energy of 1.74 keV only a small fraction of the downstream target layer contributes to the signal. The absorption length amounts to $d_{abs} = 12.244 \mu m$ [73]. In this section we describe experiments with signals from directly scattered electrons, Møller electrons and characteristic K X-rays. Details of these experiments are described in Ref. [5].

### X1.6.2 Experimental

The experimental setup is shown in Fig. X1.27. The low emittance electron beam of MAMI is well suited to prepare a beam with small angular divergence. At a beam energy of 600 MeV the horizontal and vertical emittances are $\varepsilon_h = 3.0 \pi \text{ nm rad}$ and $\varepsilon_v = 0.5 \pi \text{ nm rad}$, respectively. Typical spot sizes in our experiments with standard deviations of $\sigma_h = 340 \mu m$ and $\sigma_v = 180 \mu m$ [74] result in standard deviations of the beam divergences of $\sigma_{h} = 8.8 \mu \text{ rad}$ and $\sigma_{v} = 2.8 \mu \text{ rad}$, respectively. These numbers correspond at an entrance angle $\psi_{in} = \psi_c = 0.5 \text{ mrad}$ to a spread of the transverse energy of $\delta E^{kin}_{\perp} \leq 6.2 \text{ eV (FWHM)}$. Such a spread defines reasonable well the initial energy in the transverse potential well of 79 eV depth [5].

As targets self supporting 10 mm×10 mm pieces of silicon single crystals were used, cut with the (100) plane parallel to the surface. Thinner targets of 7.9 $\mu m$ and 14.7 $\mu m$ were prepared by anisotropically etching on a 3" wafer (nominal thickness of 466 $\mu m$, cut with the (100) plane parallel to the surface) areas of 3 mm×3 mm thin in a 30% KOH alkaline solution at 80°C [75].
Figure X1.27: Experimental setup, top view above and side view below. Downstream the Si target the beam is deflected horizontally by the $44^\circ$ bending magnet BM1 and vertically by a $7.2^\circ$ bending magnet BM2. Just in front of the beam dump the beam spot can be monitored with a ZnS luminescent screen which is viewed by a CCD camera. The CdZnTe detector can be used to investigate photons with an energy below 100 keV which are emitted in forward direction.

The crystals were mounted on goniometers with which rotations around three axes could be performed, see Fig. X1.28: (i) by the tilt angle $\alpha$ around the $x^0$ axis of the crystal frame, (ii) by the azimuthal angle $\phi$ around the $z$ axis of the laboratory reference frame, and (iii) by the polar angle $\Theta$ around the $y^0$ axis which is the projection of the $y^0$ axis on the $xy$ plane of the laboratory reference frame.

In a distance of 730 mm from the target an intrinsic germanium detector Ge(i) is positioned. A relatively large angle of $\Phi_0 = (57 \pm 1)^\circ$ with respect to the electron beam direction, the $x$ axis, was chosen to effectively decrease the downstream target layer, viewed by the detector, to $d_{\text{eff}} = d_{\text{abs}} \cos \Phi_0 = 6.67 \mu m$, with $d_{\text{abs}} = 12.244 \mu m$ [73]. The Ge(i) with a 0.4 $\mu m$ thick polymer entrance window is well suited for the detection of silicon K X-rays of 1.74 keV energy and Möller electrons. Typical spectra of the Ge(i) detector are depicted in Fig. X1.29. View graph (a) shows the spectral distribution of Möller electrons. In view graphs (b) and (c) the Möller electrons appear as a continuum background and as the overflow peak which contains all events with energies greater than 40 keV. View graph (c), taken with the crystal tuned into axial channeling, demonstrates the increase of the count rate of the characteristic silicon K X-ray line and the Möller electron overflow peak in comparison to view graph (b) which was taken off-channeling.

A signal which is also sensitive to channeling was derived from the ionization chamber behind the bending magnet BM2, see Fig. X1.27. The ionization chamber is sensitive to charged particles of electromagnetic showers which are produced by electrons leaving the nominal beam.
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Figure X1.28: (a) Top view on the target setup with definition of angles and coordinate systems. The crystals can be aligned by a three-axis goniometer. Without any rotations the crystal frame (primed coordinates) coincides with the laboratory reference frame (unprimed coordinates). (b) View to the silicon single crystal in direction of the $<100>$ strings which coincides with the $x'$ axis. The tilt angle $\alpha$, azimuthal angle $\phi$, and polar angle $\theta$ are shown also. The intrinsic ultra low energy germanium detector Ge(i) (Canberra GUL0035P) with 6.18 mm diameter and 5 mm thickness is positioned at a distance of 730 mm from the center of the target at an angle $\Phi_0 = 90^\circ - 33^\circ = 57^\circ$ with respect to the $x$ axis.

Figure X1.29: Spectra taken with the Ge(i) detector in the geometry of Fig. X1.28 (a). Spectrum shown in view graph (a) was recorded for 432 s at low gain of the amplifier with a 43.4 $\mu$m thick silicon single crystal target and an electron beam current of 0.82 nA, spectra (b) and (c) for 200 s at high gain with a 14.7 $\mu$m thick silicon single crystal and an electron beam current of 0.86 nA. View graph (b) for off channeling, and (c) in $<100>$ axial channeling. The pulse generator feeds signals with a repetition rate of 121 Hz into the preamplifier for the purpose of dead-time correction and normalization.

direction due to scattering or after emission of a multi-MeV photons in the silicon target crystal. Scans of the azimuthal angle $\phi$ for the characteristic K X-rays, the Møller electron and the ionization chamber signals are depicted in Fig. X1.30.

For the Si K X-ray and Møller electron signals the data were fitted with two Gaussians, one with positive amplitude and a narrow width $\sigma_S(t)$, the other with a negative amplitude and a larger width $\sigma_B(t)$, and a smooth function which describes the background. The ionization
Figure X1.30: Azimuthal angle \( \phi \) scans for \(< 100 >\) axial channeling of 600 MeV electrons in silicon single crystals of various thicknesses as indicated in the view graphs. Panels in the left column K show the characteristic Si K X-ray signal, in the middle column M the Möller electron signal (overflow peak in Fig. X1.29), and in the right column Io the ionization chamber signal. Data have been normalized for K X-rays and Möller electrons to 1.6 nC integrated beam charge impinging on the target, and for the ionization chamber voltage signal to a beam current of 1 nA. Error bars represent measurements, full lines best fits.

Chamber signals were fitted with only one Lorentzian and a smooth background function. The best fits are shown in Fig. X1.30 as full lines. To obtain the normalized excess count rates \( S^{(e)} \), correction factors \( \exp(\theta_0^2(t)/(2\sigma_0^2(t))) \) were applied which take into account a finite polar angle \( \theta_0 \) in the azimuthal \( \phi \) scans. For the ionization chamber signals an analogue procedure was applied with a Lorentzian. The results for \( S^{(e)} \) are shown in Fig. X1.31, row (b), as error bars.

The final aim of our measurements is to determine the dechanneling function \( F(x) \) for \(< 100 >\) axial channeling. It describes the fraction of electrons remaining in the channel after passage of a distance \( x \) in the target. The dechanneling function is proportional to the differential signal yield \( Y^{(e)}(x) = dy^{(e)}/dx \) which is the signal \( dy^{(e)} \) per unit path length \( dx \). This quantity is connected to the normalized axial channeling excess count rate \( S^{(e)} \) by the convolution integral

\[
S^{(e)}(t) = \int_0^t Y^{(e)}(x) \cdot g^{(e)}(t-x) dx
\]

(X1.9)

with \( g^{(e)}(t-x) \) a function which describes the response of the detector to a signal generated in a distance \( x \) from the target entrance interface. Fig. X1.31 depicts the results. Row (a) shows the normalized background count rate \( I^{(e)}_{a,0} \), row (b) the normalized axial channeling excess count rates \( S^{(e)} \), row (c) the initial differential yield ratios \( Y^{(e)}(t)/Y_{a,0}^{(e)} \), row (d) the widths (FWHM)
Figure X1.31: Results, extracted from the φ scans of Fig. X1.30, for the characteristic Si K X-ray signals (K, left), the Møller electron signals (M, middle), and the ionization chamber signals (Io, right) as function of the target thickness \( t \). Error bars are measurements, full lines best fit results. Data have been normalized to 1.6 nC integrated beam charge impinging on the target for K X-rays and Møller electrons, and to a beam current of 1 nA for the ionization chamber voltage signal.

of the axial channeling peaks, and row (d), the widths (FWHM) of the axial channeling peaks. Error bars are measurements, full lines calculated widths (FWHM) of the multiple scattering distributions in amorphous matter according to [76] with a fraction of 97 % in the sample.

X1.6.3 Discussion

A number of features become apparent in Fig. X1.30 and Fig. X1.31 which will be discussed in the following.
In Fig. X1.30 a depression of the yields is visible for the Si K X-ray and Møller electron signals at azimuth angles around 0.8 mrad $< \phi < 1.5$ mrad, and corresponding negative angles, which depends on the target thickness. This angular range coincides with the transition region between planar channeling of the electrons in the horizontal (001) planes and axial $< 100 >$ strings of the crystal. The suppression could originate from a chaotic transverse above barrier motion of the electrons in the crystal [77]. The K X-ray yield may approach in this region the reference value for amorphous matter to which the excess yield at axial channeling must be determined.

The suppression of the K X-ray yield around 0.8 mrad $< \phi < 1.5$ mrad is absent for the ionization chamber signals. The reason might be found in the fact that the energy loss by radiation of a photon with an energy between 4 and 10 MeV originates essentially from above barrier string scattering rather than from channeling which probably leads predominantly to the emission of photons with energies greater than 10 MeV. Electrons with an energy loss less than 4 MeV and larger than 10 MeV do not contribute to the signal, see Ref. [5].

Next the angular signal widths as function of the target thicknesses will be discussed. While for the small target thicknesses of 7.9 $\mu$m and 14.7 $\mu$m all three widths are approximately the same, they significantly differ for the largest crystal thickness of 199.5 $\mu$m. The smaller width for the Møller electron signal in comparison with the Si K X-ray signal can be explained as follows. To the Si K X-ray signal only photons contribute which are created directly at the target exit interface while high energy Møller electron may reach the detector from everywhere in the target. The Møller electron signal, therefore, is an averaged signal over the whole target thickness which should be narrower than the contribution from only the exit layer. The same is true, of course, for the ionization chamber signals. However, for the latter much broader signals are observed at large target thicknesses, see Fig. X1.31 (d). A possible explanation of this observation could be that in above barrier string scattering processes, or quasi-channeling, photon emission with energies between 4-10 MeV is much more efficient as the emission of Møller electrons with an energy larger than 40 keV (overflow peak), i.e. their impact parameter dependencies differ significantly. The results of our experiment suggest that the ionization chamber signal is more sensitive to quasi-channeling while the Møller electron signal more to the real axial channeling itself.

As shown in Fig. X1.26, the K X-ray signal predominantly originates from small impact parameters and therefore is sensitive to axial channeling. Therefore, the differential yield ratio $Y^{(K)}(t)/Y^{(K)}_{a,0}$ shown in Fig. X1.31 (c) should be proportional to the electron-dechanneling function $F(t)$, i.e. the function which describes the fraction of electrons remaining in the channel after passage of a thickness $t$ in the crystal. The ratio $Y^{(K)}(t)/Y^{(K)}_{a,0}$ features a sharp decrease to about 40 % within 19 $\mu$m and a rather flat tail extending to thicknesses beyond 200 $\mu$m. This behavior agrees qualitatively with calculations of Telegin and Khokonov on the basis of a description of the channeling process with distribution functions satisfying kinetic equations of the Fokker-Planck type [78].

The most striking feature of our experiment is, however, that at a thickness of 200 $\mu$m a fraction of about 5-10 % of the electrons obviously moves in the $< 100 >$ axial channel. The interesting question addressed next is whether the signal at deep thicknesses originates from electrons which never left the axial channel or electrons which, after the rapid dechanneling at small depths $x < 19$ $\mu$m, are recaptured into an channel after they have found in multiple
scattering processes proper initial conditions with $\psi < \psi_c$ again. In Fig. X1.32 such a possibility is schematically shown by trajectory 1. After recapture the dechanneling-rechanneling process may be repeated. Rechanneling after multiple scattering may indeed be possible. This can be concluded from the signal as function of the crystal thickness of such electrons which impinge the crystals with angles $\psi_{in}$ greater than the critical angle $\psi_c$. As can be seen from Fig. X1.30, the K X-ray signal increases, as an example, for $\psi_{in} = \psi_c$ from zero at the smallest target thicknesses to clearly a finite value at the largest thickness. A trajectory belonging to such a process is also shown in Fig. X1.32, numbered with 2. It is also interesting to notice that in Fig. X1.31 (d) the angular signal width of the K X-ray signal at larger target thicknesses agrees within the error bars with the calculated one for amorphous matter. This behavior supports also the picture of a rapid dechanneling-rechanneling process with probably a mean path length of the electron outside the channel larger than the dechanneling length. In other words, small channeling trajectories are interrupted by longer ones in which the electron scatters more or less randomly with strings or atoms of the crystal.

X1.6.4 Conclusions

Channeling experiments were performed with silicon single crystals of thicknesses between 7.9 and 200 $\mu$m at the Mainz Microtron MAMI with a low emittance electron beam of 600 MeV energy. The depth dependencies of the axial $\langle 100 \rangle$ channeling process was studied, taking advantage for the first time of Si-K characteristic X-rays with an energy of 1.74 keV and an effective projected absorption length of 6.67 $\mu$m, Möller electrons, and a signal from an ionization chamber located downstream the beam line. Results derived from K X-rays, the only signal which has been fully understood in our experiments, show that axial channeling is still observable in a depth of 200 $\mu$m. However, the angular spread of the signal as function of the crystal thickness suggests that the signal at such large depths originates from rapid dechanneling-rechanneling processes.
X1.7 Production of Extreme Infrared Radiation at the 3.41 MeV Electron Injector LINAC

X1.8 Introduction

Smith-Purcell (SP) radiation is generated when a beam of charged particles passes close to the surface of a periodic structure, i.e., a diffraction grating [79, 80, 81]. Soon after the discovery, potential applications of the SP effect became the topic of interest. In a number of theoretical and experimental studies the SP effect has been discussed as a basis for free electron lasers, for particle acceleration, or for particle beam diagnostics, for references see also [81]. The wavelength of the radiation covers the region from the visible down to the extreme infrared, the latter defined as the wavelength region from 15 $\mu$m to 1 mm. This spectral range attracts currently much attention in the so-called THz gap ($\lambda = 300 \mu$m) for many applications in the field of physics, chemistry, biology and medicine. Of particular interest are chemical and biological agent detection and the development of imaging systems for security applications. However, no efficient radiation sources like lasers or electronic devices are currently available in this spectral region.

In the last years SP-radiation from diffraction gratings has been investigated in the THz gap at the injector LINAC of MAMI [82, 83]. In these experiments the electron beam was guided close to the diffraction grating over a distance of about 20 cm by means of the magnetic field of a 5 T superconducting solenoid. In these experiments it was observed that the helical motion of the electrons in the magnetic field leads to additional synchrotron radiation emission with comparable wavelengths. Coherence effects caused by the short electron bunch length in the order of the emitted wavelength increase dramatically the emitted power. Therefore, the characteristics of the synchrotron radiation emission was investigated in a dedicated experiment [84]. The experiments and results are described in the following.

X1.8.1 Basic background

Relativistic electrons move in a homogenous magnetic field $B$ on a helical trajectories which may be characterized by the pitch angle $\alpha$ and the projected radius $r_0(\alpha) = (\beta c/\omega_H) \cos \alpha$ of the helix, see Fig. X1.33. Here $\omega_H = eB/(\gamma m_e)$ is the cyclotron frequency, $c$ the speed of light, $\beta = v/c$, and $m_e$ the rest mass of the electron. The power spectrum of the emitted synchrotron radiation per unit solid angle $d\Omega$ and unit angular frequency interval $d\omega$ can be written as [86]

$$\frac{d^2P}{d\omega d\Omega} = I \frac{e^2 \nu_0 N^2}{\omega H 4\pi} \sum_{n=-\infty}^{\infty} \frac{\sin^2 \nu_n}{\nu_n^2} F(\alpha, \theta, E, B, n)$$

where $\mu_0$ is the permeability of free space, $N$ the number of helical periods, $\theta$ the emission angle of radiation as introduced in Fig. X1.33 (a), and $\nu_n = N \pi [(1 - \beta \| \cos \theta) / \omega_H - n]$. The spectrum consists of a series of peaks at wavelengths

$$\lambda_n = \frac{2\pi c}{\omega_H n}(1 - \beta \sin \alpha \cos \theta).$$

The function $F(\alpha, \theta, E, B, n)$ describes the intensity of the $n^{th}$ harmonics. Characteristic features of the emitted radiation are: (i) in forward direction only the first harmonic is emitted, (ii)
the spectral width is inversely proportional to the number of helical periods \( N \), (iii) the angular width is inversely proportional to the relativistic factor \( \gamma \), and (iv) the larger the pitch angle \( \alpha \) is the shorter the wavelength \( \lambda \) of the emitted radiation. If the electron bunch length is in the order of the emitted wavelength coherence effects have to be taken into account. The power spectrum, integrated over the frequency, modifies to

\[
\frac{dP}{d\Omega}_{coh} = \frac{dW}{d\Omega} I \left[ 1 + \left( \frac{I}{e f_b D_f} - 1 \right) |\tilde{S}(\lambda)|^2 \right]
\]

(X1.12)

with \( \tilde{S}(\lambda) \) the longitudinal bunch form factor, \( f_b = 2.45 \text{ GHz} \) the bunch frequency and \( D_f = 0.5 \) the duty cycle of the electron beam. As an example, the bunch form factor of \( N_e \) electrons distributed according to a Gaussian with a standard deviations \( \sigma_z \) in longitudinal \( z \) direction is given by

\[
|\tilde{S}(\lambda)|^2 = \exp(-4\pi^2 \sigma^2 / \lambda^2).
\]

The calculated radiated power as function of the pitch angle \( \alpha \) is depicted in Fig. X1.34. Due to the coherence effects the radiation is strongly enhanced at pitch angles less than 80° corresponding to wavelengths larger than 0.5 mm of the first harmonics. The small peak at a wavelength of 200 \( \mu \text{m} \) results from incoherent radiation emission. In the next subsection an experiment is described which was performed to measure the power spectrum and to determine the bunch distribution of the electrons from the bunch form factor.

**X1.8.2 Experimental**

The experimental setup is shown in Fig. X1.35. The electron beam is guided with vertical and horizontal steerer magnets into the superconducting solenoid. The infrared radiation is focused into the opening aperture of the bolometer with a diameter of 11 mm. The electron beam is chopped with a frequency of 14.9 Hz. Detector noise is reduced by employing the lock-in technique. The wavelength of the radiation can be selected by band pass filters with center wavelengths of 150 \( \mu \text{m} \), 350 \( \mu \text{m} \), and 1250 \( \mu \text{m} \). The radius of the helical motion can be measured by inspection of the electron beam on a fluorescence screen positioned 760 mm.
Figure X1.34: Calculated power spectrum as function of the pitch angle $\alpha$. The corresponding wavelength of the first harmonics is given in the upper scale. Coherence effects were taken into account by equation (X1.12) assuming a Gaussian with standard deviation $\sigma_z = 250 \mu$m for the electron distribution in the bunch. Electron beam energy $E = 3.41$ MeV, magnetic field $B = 4.6$ T, length of the homogeneous part of the magnetic field $L = 300$ mm, mean beam current $I = 10$ \mu A, and observation angle $\theta = 0.1^\circ$.}

downstream from center of the solenoid in a low magnetic field $B$. If the magnetic field is ramped by $\pm 0.1$ T around the nominal field $B_0 = 4$ T the electron beam gyrates with the radius $r$ on the fluorescence screen. Assuming that the adiabatic invariance [85] holds, the radius $r_0$ of the helix in the solenoidal field $B_0$ can be calculated by the equation $r_0 = r \sqrt{B/B_0}$.

### X1.8.3 Measurements and results

The power of the emitted radiation was investigated as function of the pitch angle $\alpha$ and wavelength $\lambda$. Typical spectra are shown in Fig. X1.36. The decrease of the power at smaller pitch angles $\alpha$ in view graphs (b) and (c) originates from the transmission characteristic of the filters. For a quantitative comparison with calculations according to equation (X1.12) a rectangular, Gaussian, and a double humped Gaussian shape were assumed for the longitudinal distribution of the electrons in the bunch. Best agreement was found for the latter with a half width of 240 $\mu$m. The corresponding calculations are also shown in Fig. X1.36 as dashed lines. Only for the measurement without a wavelength filter the absolute value of the radiated power agrees with the measurements. The disagreement between measurement and calculation may originate from a bunch length increase if the beam current is enlarged. As an example, already an increase of the bunch length to 320 $\mu$m results in a drastic reduction of the bunch form factor at shorter wavelengths and consequently also a reduction of the transmitted power through the filters at the specified wavelengths.

Because of this conjecture, a dedicated measurement of the bunch length has been restricted to electron beam currents less than 0.6 \mu A. The results of the measurement are shown in Fig. X1.37. A quadratic increase of the power is observed for both filters with center wavelengths of 150 and 350 $\mu$m. Both measurements can be consistently explained by a bunch length of about 350 $\mu$m (FWHM) [84]. However, strong interference effects in the bunch form factor as
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Figure X1.35: Experimental setup. The pitch angle $\alpha$ of the electron beam in the 5 T superconducting solenoid can be adjusted with the steerer magnet in front of chamber 1. The magnetic field is sufficiently homogeneous over a length of $L = 300$ mm. In vacuum chamber 2 the radiation is coupled out through a 2 mm polyethylene window after reflection with the aid of an aluminized plastic foil which has a hole for the electron beam. The spherical mirror outside the vacuum system focuses the radiation into the Winston cone of a liquid helium cooled composite silicon bolometer which can be moved horizontally and vertically by translation stages. The optical system accepts a radiation cone with an opening angle $\theta_{\text{max}} = 3^\circ$. The electron beam can be monitored with a fluorescence screen.

Figure X1.36: Measured power as function of the pitch angle $\alpha$ (lower scale) or the wave length of the corresponding first harmonics $\lambda$ of the first harmonics according to equation (X1.11) (upper scale). Full line: power spectra for filters with central wavelength as indicated. Dashed line: calculations according to equation (X1.12) assuming a double humped electron bunch distribution with a length of 240 $\mu$m (FWHM). Notice, that in view graph (b) and (c) the calculated intensities were scaled down by a factor of 3 and 50, respectively.

function of the bunch length, which sensitively depend on the assumed shape of the chosen bunch distribution function, indicate a strong model dependence of the extracted bunch length.

X1.8.4 Discussion

The radiated power of the synchrotron radiation in the extreme infrared spectral range due to the helical motion of electrons in the strong magnetic field of 4 T is governed by the effect of
coherent emission of the electron bunch. This fact enables bunch length measurements. The measured bunch length of about 350 \( \mu m \) is in accord with measurements done with the MAMI phase space monitor (Nilles).

For the Smith-Purcell experiments in progress, this kind of synchrotron radiation can be nearly completely suppressed by a proper choice of the pitch angle which must be chosen close to 90\(^\circ\). First experiments have been performed with aluminum gratings of a period length of 170 \( \mu m \). An optical mirror system was designed to detect radiation emitted at an angle around 90\(^\circ\) with respect to the grating surface. Radiation was observed with maximum intensity at a wavelength of about 600 \( \mu m \) which is at variance with what is expected from the Smith-Purcell emission characteristics. Simulation calculations have revealed that this strong radiation component must be emitted at the downstream edge of the grating in forward direction. The investigation of this novel type of radiation will be continued with a detection system optimized for radiation which is emitted in forward direction.


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CHAPTER 2. EXPERIMENTS AT MAMI AND THEORY


